Math 324 D | Quiz 3 (20 minutes) | July 31, 2015

Name: Key

> Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. Leave your answers in *exact form* instead of decimal approximations.

1. Let $\vec{F} = (3x^2e^{2y} + \sin y)\vec{i} + (2x^3e^{2y} + x\cos y + 1)\vec{j}$ be a vector field defined on all of $\mathbb{R}^2$.

   (a) (2 points) Use the partial derivatives of the component functions to show that $\vec{F}$ is conservative.

   $\frac{\partial P}{\partial y} = 6x^2e^{2y} + \cos y = \frac{\partial Q}{\partial x}$ | Since $\mathbb{R}^2$ is simply connected, $\Rightarrow \vec{F}$ is conservative.

   (b) (1 point) Let C_1 be the unit circle. Determine the value of

   $$\oint_{C_1} \vec{F} \cdot d\vec{r}.$$

   C_1 is a closed curve, & $\vec{F}$ is conservative so $\boxed{=0}$

   (c) (4 points) Find a potential function for $\vec{F}$, (i.e., a function f such that $\vec{F} = \nabla f$).

   We know such an f exists by (a).

   $P = f_x = 3x^2e^{2y} + \sin y$ (+1)

   integrate w.r.t. x.

   $f = x^3e^{2y} + x\sin y + g(y)$ some g. (+1)

   $Q = f_y = 2x^3e^{2y} + x\cos y + g'(y)$ (+1)

   so $g'(y) = 1$

   $\Rightarrow g(y) = y + K$ (+1)

   So $f(x,y) = x^3e^{2y} + x\sin y + y + K$ works $\forall K \in \mathbb{R}$.

   (d) (3 points) Let C_2 be a curve given by $\vec{r}(t) = \cos t\vec{i} + \frac{1}{2}t\vec{j}$ for $0 \le t \le \pi$. Compute

   $$\int_{C_2} \vec{F} \cdot d\vec{r}.$$

   Fundamental theorem of line integrals:

   $\int_{C_2} \vec{F} \cdot d\vec{r} = f(\vec{r}(\pi)) - f(\vec{r}(0))$.

   $\vec{r}(\pi) = \langle -1, \frac{\pi}{2} \rangle$

   $\vec{r}(0) = \langle 1, 0 \rangle$

   w/ $K=0$

   $f(-1, \frac{\pi}{2}) = (-1)^3e^{\pi} + (-1)(1) + \frac{\pi}{2}$

   $= -e^{\pi} - 1 + \frac{\pi}{2}$

   $f(1,0) = 1$

   So $\int_{C_2} \vec{F} \cdot d\vec{r} = \boxed{-e^{\pi} - 2 + \frac{\pi}{2}}$