Key

Math 324 D   Quiz 1 (20 minutes)   June 26, 2015

Name:

> Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. Leave your answers in *exact form* instead of decimal approximations.

1. (5 points) Compute the following integral:

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{\cos x^2}{x} dxdy.$$

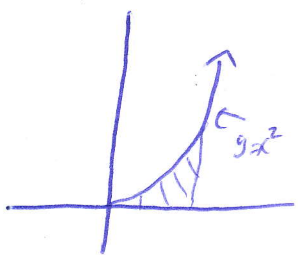
First we reverse order

Sketch Region

$\{0 \le y \le 1, \sqrt{y} \le x \le 1\}$

$\quad\quad ||$

$\{0 \le x \le 1, 0 \le y \le x^2\}$



Then integrate

$= \int_0^1 \int_0^{x^2} \frac{\cos x^2}{x} dy\, dx$

$= \int_0^1 x \cos x^2 dx$ $\quad\Big|\; u = x^2,\; du = 2x dx,$ bounds, $u: 0 \to 1$

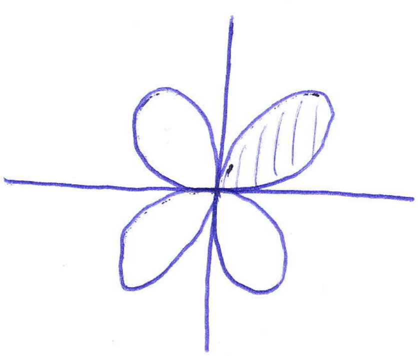
$= \frac{1}{2} \int_0^1 \cos u\, du$

$= \frac{1}{2} (\sin u)_0^1 = \boxed{\frac{\sin 1}{2}}$

2. (5 points) The function $r = \sin 2\theta$ in polar coordinates is often called a polar rose due to it's flower-like shape. Sketch the region and compute the area of one petal.

$r = 0$ at $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$

So



$(*)$ $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$

$= 1 - 2\sin^2 2\theta$

$\sin^2 2\theta = \frac{-\cos 4\theta + 1}{2}$

1 petal is first quadrant.

Area is $\iint_D 1 dA$. So

$A = \int_0^{\pi/2} \int_0^{\sin 2\theta} r\, dr\, d\theta$

$= \frac{1}{2} \int_0^{\pi/2} \sin^2 2\theta\, d\theta \; (*)$

$= \frac{1}{4} \int_0^{\pi/2} (1 - \cos 4\theta) d\theta$

$= \frac{1}{4} \left[\theta - \sin 4\theta\right]_0^{\pi/2}$

$= \frac{1}{4} \left[\frac{\pi}{2}\right] = \boxed{\frac{\pi}{8}}$