Changing order of integration "1 by 1"

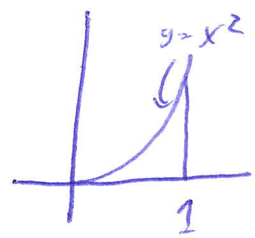
We have

$$\int_0^1 \int_0^{x^2} \underbrace{\int_0^y f(x,y,z)\,dz}\,dy\,dx$$

This is a function of x and y. Call it $g(x,y)$.

$$= \int_0^1 \int_0^{x^2} g(x,y)\,dy\,dx$$

Reverse as usual



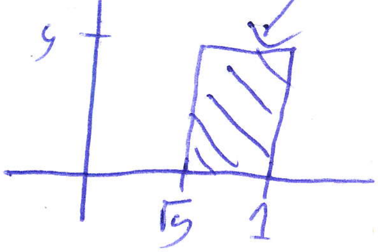
$$= \int_0^1 \int_{\sqrt{y}}^{1} g(x,y)\,dx\,dy$$

replace g $= \displaystyle\int_0^1 \underbrace{\left[\int_{\sqrt{y}}^1 \int_0^y f(x,y,z)\,dz\,dx\right]}\,dy = \int_0^1 h(y)\,dy = (**)$

This is a function of y. Call it $h(y)$.

Look at $h(y) = \displaystyle\int_{\sqrt{y}}^{1} \int_0^y f(x,y,z)\,dz\,dx$

For a fixed $y \in [0,1]$ our domain is:



A RECTANGLE So reverse for free

$$h(y) = \int_0^y \int_{\sqrt{y}}^1 f(x,y,z)\,dx\,dz$$

$$\text{So } (**) = \int_0^1 \int_0^y \int_{\sqrt{y}}^1 f(x,y,z)\,dx\,dz\,dy$$