Math 124 KAUST Midterm 2 (80 minutes) February 27, 2017

Name: Key

Directions:

- You have 80 minutes to complete this exam.
- Only TI 30 Calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

Question	Points	Score
1	15	
2	20	
3	10	
4	10	
5	25	
Total:	80	

1. Answer the following true or false questions. You do not need to justify your work.

   (a) (3 points) If f is a smooth function, and $f(2) = f(8)$, then there is some number c, with $2 < c < 8$, so that $f'(c) = 0$.

   True | Rolle's Theorem

   (b) (3 points) Suppose f is a function with $f'(15) = 0$. Then f has either a local maximum or a local minimum at 15.

   False | e.g. $f(x) = (x-15)^3$

   (c) (3 points) Let f be a continuous function defined on (a, b). Then f has an absolute maximum in the interval (a, b).

   False | e.g. $f(x) = x$, $(a, b) = (0, 1)$

   (d) (3 points) Let f be a smooth function, with $f(0) = 0$ and $f(10) = 20$. Then there is a number between 0 and 10, whose the tangent line to f has a slope of 2.

   True | Mean Value Theorem

   (e) (3 points) A local maximum or local minimum must occur at a critical number.

   True | Fermat's Theorem

2. In each of the following, compute $\frac{dy}{dx}$. You may need to use logarithmic or implicit differentiation. There is no need to simplify your answers.

(a) (5 points)

$$y = \frac{(\sin x)^{\sqrt{x}}}{(\sqrt{x})^{\sin x}}.$$

quotient rule -1

$\ln y = \sqrt{x}\ln(\sin x) - \sin x \ln(\sqrt{x})$

$\ln y = \sqrt{x}\ln(\sin x) - \frac{1}{2}\sin x \ln(x)$

miscopied formula: -2

$\frac{y'}{y} = \left(\frac{\ln(\sin x)}{2\sqrt{x}} + \frac{\sqrt{x}\cos x}{\sin x} - \left(\frac{1}{2}\cos x \ln x + \frac{\sin x}{2x}\right)\right)$ (*)

$\ln(\frac{a}{b}) = \frac{\ln a}{\ln b}$

-1

$\frac{y'}{y}$ missing

$y' = \frac{(\sin x)^{\sqrt{x}}}{(\sqrt{x})^{\sin x}} \cdot \left(* \right)$

(b) (5 points)

$$\sin(xy^2) = x^5 + y^6.$$

$\cos(xy^2)\cdot(y^2 + 2xy\cdot y') = 5x^4 + 6y^5\cdot y'$

Algebra errors -1 each

$\cos(xy^2)2xy\cdot y' - 6y^5\cdot y' = 5x^4 - \cos(xy^2)\cdot y^2$

$y'\left(\cos(xy^2)2xy - 6y^5\right) = 5x^4 - \cos(xy^2)y^2$

missing product rule

-1

$y' = \frac{5x^4 - y^2\cos(xy^2)}{2xy\cos(xy^2) - 6y^5}$

(c) (5 points)

$$y = \ln(\ln(5^x)).$$

missing -1

$$y' = \frac{1}{\ln(5^x)} \cdot \frac{1}{5^x} \cdot 5^x \ln(5)$$ ← ok here

$$= \frac{1}{x \ln 5} \cdot \frac{5^x}{5^x} \cdot \ln(5)$$

$$= \frac{1}{x}$$

$5^x \mapsto x 5^{x-1}$

-2

(d) (5 points)

$$y = \arcsin(2x) \cdot \arccos\left(3x^2\right).$$

$$y' = \frac{\arccos(3x^2)}{\sqrt{1-(2x)^2}} \cdot 2 + \frac{-\arcsin(2x)}{\sqrt{1-(3x^2)^2}} \cdot 6x$$

no product rule?

-2

/

3. (10 points) Use linear approximation to estimate $\sqrt[7]{129}$, by linearizing the function $f(x) = \sqrt[7]{x}$ at a suitable number. (HINT: What number near 129 do you know 7th root of?).

(+2) Let $a = 128$

$a = 2^7$
-2

(+1) Then $f(a) = \sqrt[7]{128} = 2$.

~~Linearize~~ $f'(x) = \frac{1}{7} \cdot x^{-6/7}$

(+2) $f'(a) = \frac{1}{7} \cdot \sqrt[7]{128}^{-6} = \left(\frac{2^{-6}}{7}\right) = \frac{1}{7 \cdot 64} = \frac{1}{448}$

Linearize f about a.

(+3) $L(x) = f(a) + f'(a)(x-a)$

$= 2 + \frac{1}{448}(x - 128)$

$\sqrt[7]{129} = f(129) \approx L(129) = 2 + \frac{1}{448}(129 - 128)$

(+2)

$= 2 + \frac{1}{448} = \frac{897}{448}$

4. (10 points) Let $f(x) = 2x^3 - 6x + 5$. Find the absolute maximum and minimum values of f on the interval $[-2.5, 1.5]$.

Find critical points

$f'(x) = 6x^2 - 6$

$f'(x) = 0, \quad x = \pm 1$

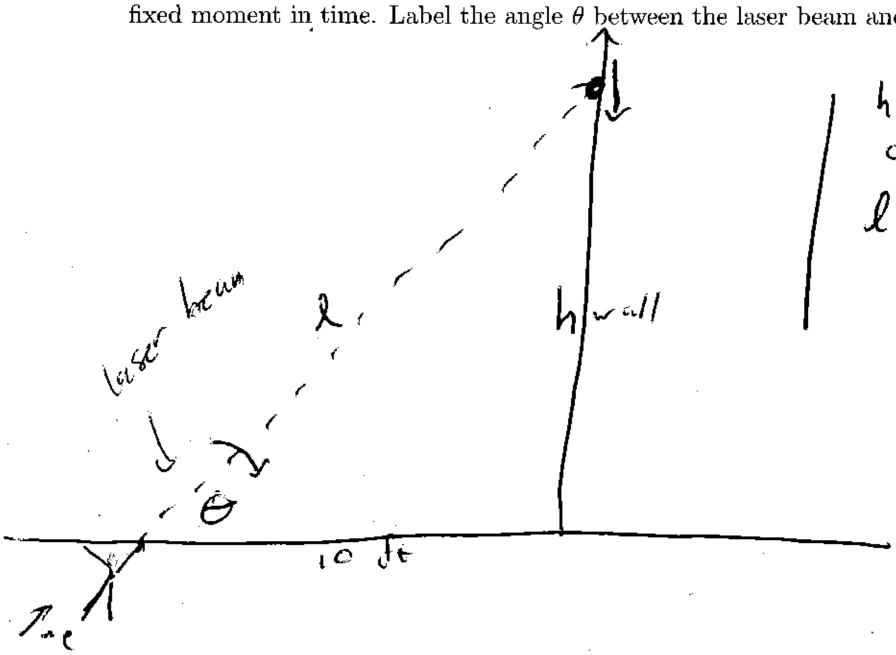
(+4)

Candidates

	x	$f(x)$	
(+2) endpoints	-2.5	-11.25	← min
	1.5	2.75	
(+2) critical points	$+1$	1	(+2)
	-1	9	← max

5. You stand ten feet away from an infinitely tall wall, holding a high powered laser pointer. You point the laser beam straight up, and at a constant angular velocity lower it. After 10 seconds it is pointed horizontally, directly at the wall.

(a) (5 points) Draw a picture of the situation. Label yourself, the wall, and draw the laser beam at a fixed moment in time. Label the angle θ between the laser beam and a horizontal line.



h = height of beam on wall

l = length of beam.

(b) (10 points) Compute the speed at which the end of the beam is traveling along the wall when it is at an angle of $\theta = \pi/3$ radians above horizontal.

Unknown

(+1) $\frac{dh}{dt}$ = speed at which the beam is moving along wall

Known

(+2) $\frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2 \text{ rad}}{10 \text{ sec}}$

$= \frac{\pi}{20}$

(should be <0)

Relate θ & h. (+3)

$\tan\theta = \frac{h}{10}$

so $h = 10\tan\theta$

differentiate wrt t. (+4)

(apply $\frac{d}{dt}$)

$\frac{dh}{dt} = 10\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{\pi}{2}\sec^2\theta$

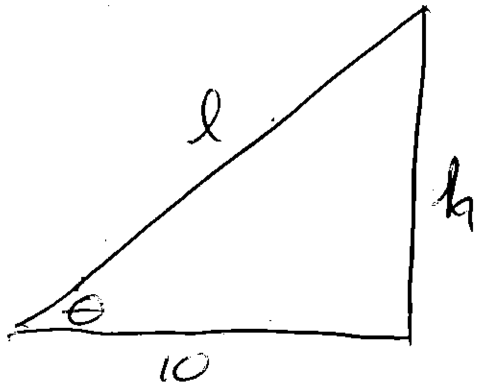
$\left.\frac{dh}{dt}\right|_{\theta=\frac{\pi}{3}} = \frac{\pi}{2}\sec^2\theta = \frac{\pi}{2}\cdot 4 = 2\pi$

$\boxed{2\pi \ \frac{ft}{sec}}$

speed pos, fine

no chain rule? −2

(c) (5 points) Find an equation for the length of the laser beam in terms of θ.



$\cos\theta = \frac{10}{l}$

$l = 10(\cos\theta)^{-1} = 10\sec\theta$

$\nwarrow$ not $\underline{\cos^{-1}\theta}$

(d) (5 points) Compute the rate at which the length of the laser beam is decreasing when $\theta = \pi/3$ radians above horizontal.

Differentiate wrt t -

$\frac{dl}{dt} = 10 \tan\theta\sec\theta \frac{d\theta}{dt} = \frac{\pi}{2}\tan\theta\sec\theta$

$\sec(\pi/3) = 2$

$\tan(\pi/3) = \sqrt{3}$

so $\left.\frac{dl}{dt}\right|_{\theta=\pi/3} = \frac{\pi\sqrt{3}}{m}$ $\frac{ft}{sec}$

No chain rule

-2

—or—

$10^2 + h^2 = l^2$

so $hh' = ll'$

$l' = \frac{hh'}{l}$

h' from a wrong h'

-1