Math 124 KAUST Midterm 1 (80 minutes) February 2, 2017

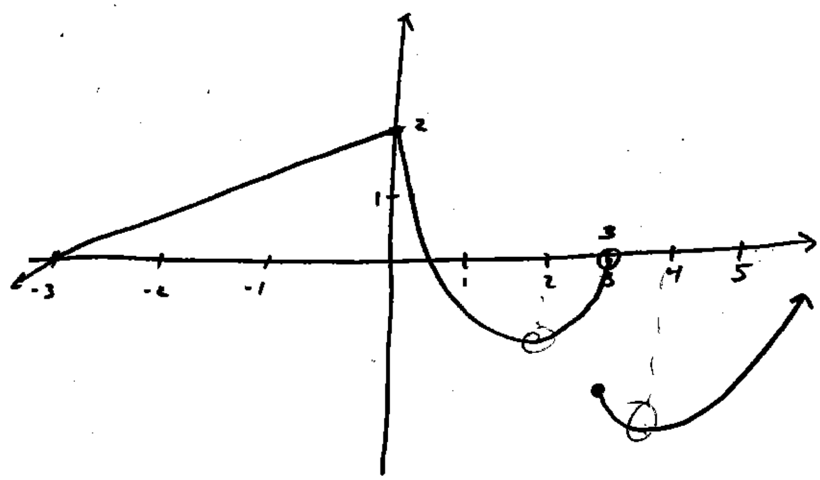
Name: Key

Directions:

- You have 80 minutes to complete this exam.
- Only TI 30 Calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

Question	Points	Score
1	15	
2	25	
3	30	
4	10	
5	10	
Total:	90	

1. The following is a graph of the function $f(x)$.



(a) (3 points) For which values of x does $f'(x) = 0$?

$x \approx 2$

$x \approx 4$

(b) (3 points) On which intervals is $f'(x) < 0$? Be sure to specify whether the intervals are open or closed (i.e., do they include their endpoints?).

$6 < x < 2$

$3 < x < 4$

(c) (3 points) At which points is f *not* differentiable?

$x = 0,\quad x = 3$

(d) (3 points) What is $f'(-2)$?

$\frac{\Delta y}{\Delta x} = \frac{2}{3}$

(e) (3 points) Compute $\lim_{x \to 3^-} f(x)$.

0

2. Compute the following limits. Show and justify all steps!

(a) (5 points)

$$\lim_{x\to-\pi}\frac{\frac{1}{\pi}+\frac{1}{x}}{x+\pi}. =$$

$$= \lim_{x\to\pi}\frac{\frac{x+\pi}{x\pi}}{x+\pi} \quad (+2)$$

$$= \lim_{x\to-\pi}\frac{1}{x\pi} = \frac{-1}{\pi^2} \quad (+1)\ (+2)$$

(b) (5 points)

$$\lim_{u\to1}\frac{\sqrt{7u-3}-2}{u-1}. \cdot \frac{\sqrt{7u-3}+2}{\sqrt{7u-3}+2} \quad (+2)$$

$$= \lim_{u\to1}\frac{7u-7}{u-1(\sqrt{7u-3}+2)} \quad (+2)$$

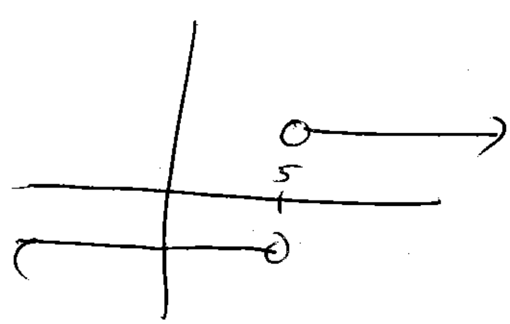
$$= \lim_{u\to1}\frac{7}{(\sqrt{7u-3}+2)} = \frac{7}{4} \quad (+1)$$

(c) (5 points)

$$\lim_{t\to5}\frac{t-5}{|t-5|}.$$

$$\lim_{t\to5^-}\frac{t-5}{|t-5|} = \lim_{t\to5^-}\frac{t-5}{5-t} = -1 \quad (+2)$$

$$\lim_{t\to5^+}\frac{t-5}{|t-5|} = \lim_{t\to5^+}\frac{t-5}{t-5} = 1 \quad (+2)$$



limite DNE (+1)

(d) (5 points)

$$\lim_{z\to\infty} \frac{2z^2+5z+7}{7z^3+3} \cdot \frac{1/z^3}{1/z^3}$$ +2

$$= \lim_{z\to\infty} \frac{\frac{2}{z} + \frac{5}{z^2} + \frac{7}{z^3}}{7 + 3/z^3}$$

$$= \frac{0}{7} = 0$$ +2 +1

(e) (5 points)

$$\lim_{x\to-\infty} \left(x+\sqrt{x^2+7}\right) \cdot \frac{x-\sqrt{x^2+7}}{x-\sqrt{x^2+7}}$$ +2

$$= \lim_{x\to\infty} \frac{x^2-x^2-7}{x-\sqrt{x^2+7}} =$$

$$= \lim_{x\to\infty} \frac{-7}{x-\sqrt{x^2+7}} = 0$$ +1 +2

approaches $-\infty$

3. Compute the derivatives of the following functions. Show and justify all steps! Once computed, you do not need to simplify the derivative.

(a) (5 points)

$$f(x) = \sqrt[3]{1+4x}. = (1+4x)^{1/3}$$

$$f'(x) = \frac{1}{3}(1+4x)^{-2/3} \cdot 4$$

no chain rule (1)

1/5

(i.e. just inside)

no chain rule (2)

(i.e. just aside)

3/5

(b) (5 points)

$$g(x) = x^2 e^{-1/x}. = x^2 e^{-x^{-1}}$$

$$g'(x) = 2xe^{-1/x} + x^2 e^{-1/x} \cdot x^{-2}$$

$$= 2xe^{-1/x} + e^{-1/x}$$

No product rule

2/5 but chain

No product or chain

0/5

No chain rule, but product ok

2/5

(c) (5 points)

$$h(t) = \sqrt{\frac{t^2+4}{t^2+16}}. = \left(\frac{t^2+4}{t^2+16}\right)^{1/2}$$

$$h'(t) = \frac{1}{2}\left(\frac{t^2+4}{t^2+16}\right)^{-1/2}\left(\frac{2t(t^2+16) - 2t(t^2+4)}{(t^2+16)^2}\right)$$

quotient rule but no chain rule

2/5

Chain but no quotient

2/5

neither

0/5

Chain rule mistake

3/5 or 4/5

(d) (5 points)

$$w(s) = 2^{\tan(\pi s)}.$$

No chain rule (no deriv of inside) 2/5

...re but not often 3/5

power rule 0/5

(power w/ chain 1/5)

$$w'(s) = \ln 2 \cdot 2^{\tan(\pi s)} \cdot \sec^2(\pi s) \cdot \pi$$

(e) (5 points)

$$x(t) = \left(t^4 + 2t + 1\right)^3 \left(10t^3 + 5\right)^2.$$

product no chain 2/5

chain no product 2/5

neither 0/5

$$x'(t) = 3(t^4+2t+1)^2(4t^3+2)(10t^3+5)^2$$
$$+ (t^4+2t+1)^3 \cdot 2(10t^3+5) \cdot (30t^2)$$

(f) (5 points)

$$q(\theta) = \sin(\sin(\sin(\theta))).$$

~~chain rule started but no derivative~~

$$q'(\theta) = \cos(\sin(\sin\theta)) \cdot \cos(\sin\theta) \cdot \cos\theta$$

$q'(\theta) = \cos(\cos(\cos(\theta)))$ ← 0/5

$\sin \xmapsto{d} -\cos$

-1

4. A particle is moving along the x-axis according to the equation $p(t) = t^3 + 2t^2 + t + 5$. (NOTE: The domain of the time variable is all the real numbers, and thus can be both positive and negative).

(a) (3 points) Find the instantaneous velocity when $t = 1$.

$v(t) = p'(t) = 3t^2 + 4t + 1$

$v(1) = p'(1) = 3 + 4 + 1 = 8$

(b) (3 points) Find all times where the instantaneous velocity is equal to 0.

$3t^2 + 4t + 1 = 0$

$t = \dfrac{-4 \pm \sqrt{16-12}}{6} = \dfrac{-4 \pm 2}{6} = \underline{-1 \text{ or } -1/3}$

~~wrong fact~~

Sign error: $-1/2$

(c) (4 points) On what interval is the particle accelerating? (i.e., when is acceleration$>$ 0?)

$a(t) = v'(t) = 6t + 4$ (r)

$6t + 4 > 0$

$6t > -4$

$t > -2/3$

Sign error

$-1/2$

5. Consider the curve given by $y = 3x^2 + 4x + 5$.

(a) (3 points) Find the tangent line at the point $(-2, 9)$.

$y' = 6x + 4$

$m = y'(-2) = -12 + 4 = -8$

$\boxed{y = -8(x+2) + 9}$

(b) (7 points) Find all points where the tangent line through that point has a root at $x = 1$.

step 1 tangent line @ $x = a$.

$y = \underbrace{(6a+4)}_{y'(a)}(x-a) + \underbrace{3a^2 + 4a + 5}_{y(a)}$ +3

step 2 Line contains (1, 0) +4

$0 = (6a+4)(1-a) + 3a^2 + 4a + 5)$

$0 = +6a + 4 - 6a^2 - 4a + 3a^2 + 4a + 5$

$3a^2 - 6a - 9 = 0$

$3(a^2 - 2a - 3) = 0$

$3(a-3)(a+1) = 0$

$a = 3$ or $a = -1$

$f(3) = 27 + 12 + 5 = 44$

$f(-1) = 3 - 4 + 5 = 4$

2 pts

(3, 44)

(-1, 4)