Math 124 KAUST Derivatives Worksheet October 31, 2018

Name: Key

1. Warmup! Compute the derivatives of the following functions:

(a)

$$f(x) = 3x^5 + 4x^2 - 6x + 2$$

$15x^4 + 8x - 6$

(b)

$$g(x) = 2^x + 5\arcsin x + 10\ln x$$

$\ln 2 \cdot 2^x + \frac{5}{\sqrt{1+x^2}} + \frac{10}{x}$

(c)

$$h(x) = 1/x + 2/x^2 + 3/x^3.$$

$\frac{-1}{x^2} - \frac{4}{x^3} - \frac{9}{x^4}$

2. Product and Quotient Rule! Compute the derivatives of the following functions:

(a)

$$f(x) = 2^x 3^x + 10.$$

$\ln 2 \cdot 2^x \cdot 3^x + \ln 3 \cdot 2^x \cdot 3^x$

$= 2^x \cdot 3^x (\ln 2 + \ln 3)$

(b)

$$g(x) = \frac{\sin x}{\tan x + 2}.$$

$$\frac{\cos x (\tan x + 2) - \sin x (\sec^2 x)}{(\tan x + 2)^2}$$

(c)

$$h(x) = e^x x^2 \arctan(x).$$

$e^x \cdot x^2 \cdot \arctan x + 2x \cdot e^x \cdot \arctan x$

$$+ \frac{e^x \cdot x^2}{1 + x^2}$$

3. Chain Rule! Compute the following derivatives.

(a)

$$\ln(x^3 + 7x + 3).$$

$$\frac{3x^2 + 7}{x^3 + 7x + 3}$$

(b)

$$\frac{1}{\sqrt{x^3 - 3}}.$$

$$= (x^3 - 3)^{-1/2}$$

$$-\frac{1}{2}(x^3 - 3)^{-3/2} \cdot (3x^2)$$

(c)

$$\ln(\ln(\ln(\sin(x)))).$$

$$\frac{1}{\ln(\ln(\ln(\sin x)))} \cdot \frac{1}{\ln(\sin x)} \cdot \frac{1}{\sin x} \cdot \cos x$$

4. Implicit and Logarithmic Differentiation! Don't forget the product rule and chain rule!! Find $\frac{dy}{dx}$.

(a) $x^2 + 2xy + y^3 = 26.$

$$2x + 2y + 2xy' + 3y^2 y' = 0$$

$$y'(2x + 3y^2) = -2x - 2y$$

$$y' = \frac{-2x - 2y}{2x + 3y^2}$$

(b) $y = (x^3 - 3)^{\sin x}.$

$$\ln y = \sin x \ln(x^3 - 3)$$

$$\frac{y'}{y} = \cos x \ln(x^3 - 3) + \frac{3x^2}{x^3 - 3} \sin x$$

$$y' = (x^3 - 3)^{\sin x} \left(\cos x \ln(x^3 - 3) + \frac{3x^2 \sin x}{x^3 - 3} \right)$$

Putting it all together! Find $\frac{dy}{dx}$.

(a)

$$y = (2x^3 + 4x + 1)^4 \cdot (x^4 + x + 1)^5.$$

$$y' = 4(2x^3+4x+1)^3(6x^2+4)(x^4+x+1)^5$$

$$+ (2x^3+4x+1)^4 \cdot 5(x^4+x+1)^4(4x^3+1).$$

(b)

$$y = x \cdot \ln(\arcsin(x))]$$

$$y' = \ln(\arcsin(x)) + x \cdot \frac{1}{\arcsin x} \cdot \frac{1}{\sqrt{1+x^2}}$$