Math 124 KAUST Midterm 2 (80 minutes) November 21, 2018

Name: Key

Directions:

- You have 80 minutes to complete this exam.
- Only TI 30 Calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

Question	Points	Score
1	20	
2	15	
3	10	
4	10	
5	15	
Total:	70	

1. For each of the following, compute dy/dx. You may need to use implicit differentiation or logarithmic differentiation. As usual, do not forget the product rules or chain rules. You do not need to simplify your answers.

(a) (5 points)

$$y = \sin(\cos(\tan(\ln(x))))$$

$y' = \cos(\cos(\tan(\ln(x))))$
$\times (-\sin(\tan(\ln(x))))$
$\times \sec^2(\ln(x))$
$\times \frac{1}{x}$

1 pt per chain

(b) (5 points)

$$\sin^2(xy) + \cos(x^2y^2) = 1.$$

Implicit

$2\sin(xy)\cos(xy)[y + xy'] - \sin(x^2y^2)[2xy^2 + 2x^2y \cdot y'] = 0$

$xy' 2\sin(xy)\cos(xy) - 2x^2yy'(\sin(x^2y^2)) = \sin(x^2y^2)2xy^2 - 2\sin(xy)\cos(xy)\cdot y$

$$y' = \frac{2xy^2\sin(x^2y^2) - 2y\sin(xy)\cos(xy)}{2x\sin(xy)\cos(xy) - 2x^2y\sin(x^2y^2)}$$

3 pts for imp diff

2 pts solving for y'

(be gentle)

$\ln \frac{a}{b} = \ln a - \ln b$ (-1)

(c) (5 points)

$$y = \frac{(\sin(x))^{\ln x}}{(\ln x)^{\sin x}}$$

-1 for missing ln

$$\ln y = \ln x \cdot \ln(\sin(x)) - \sin(x)\ln(\ln(x))$$

$$\frac{y'}{y} = \frac{\ln \sin(x)}{x} + \ln x \cdot \frac{\cos x}{\sin x} - \cos x \ln(\ln(x)) - \sin x \cdot \frac{1/x}{\ln x}$$

$$y' = \frac{\sin(x)^{\ln x}}{\ln(x)^{\sin x}}\left[\frac{\ln \sin x}{x} + \ln x \cot x - \cos x \ln \ln x - \frac{\sin x}{x \ln x}\right]$$

(d) (5 points) If $f(12) = 4$ and $f'(12) = 10$. If $g(x) = 2^{f(x)}$, what is $g'(12)$? Explain all your reasoning.

$$g'(x) = \ln 2 \cdot 2^{f(x)} \cdot f'(x)$$

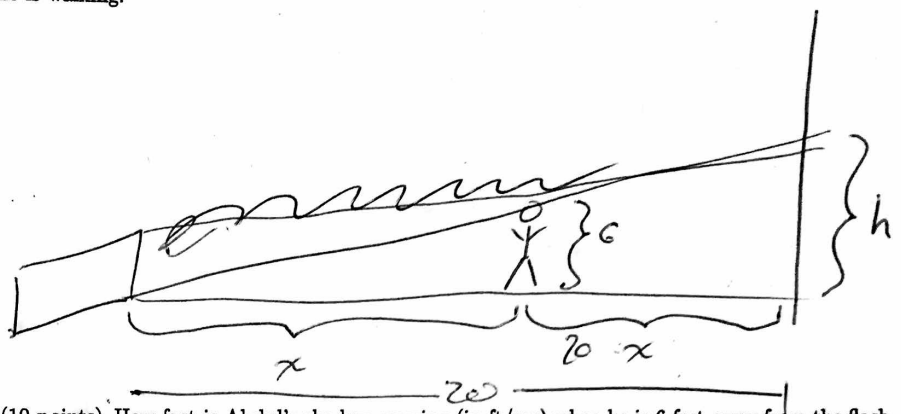
$$g'(12) = \ln(2) \cdot 2^{f(12)} \cdot f'(12)$$

$$= \ln 2 \cdot 2^4 \cdot 10$$

$$= 160 \ln 2$$

2. A spotlight is 20 feet away from a wall. Abdul is 6 feet tall, and starts walking from the wall towards the light at a speed of 2 feet per second. As he approaches the light, his shadow is projected on the wall, and it is growing.

(a) (5 points) Draw a picture of the situation. Label Abdul, the flashlight, the wall, and the direction he is walking.



(b) (10 points) How fast is Abdul's shadow growing (in ft/sec) when he is 6 feet away from the flashlight? What about when he is 1 foot from the flashlight?

Quantities

x = dist from light

h = height of shadow

$\frac{dx}{dt} = -2$

$\frac{dh}{dt} =$ WANT

Relation

$\frac{h}{20} = \frac{6}{x}$

$h \cdot x = 120$

$\frac{d}{dt}$

$x\frac{dh}{dt} + h\cdot\frac{dx}{dt} = 0$

$\frac{dh}{dt} = -\frac{h\frac{dx}{dt}}{x} = 2\frac{h}{x}$

$\cdot 2$

Missing or? (-1)

$x = 6$

$\Rightarrow \frac{h}{20} = 1$

$h = 20$

$\frac{dh}{dt} = 2\cdot\frac{20}{6} = \frac{40}{6} = \boxed{\frac{20}{3}}$

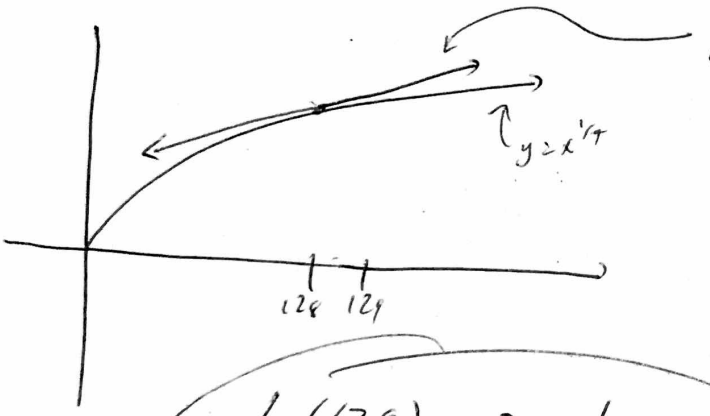
$x = 1 \quad \frac{h}{20} = 6 \quad h = 120$

$\frac{dh}{dt} = 2\cdot\frac{120}{1} = \boxed{240}$

3. (10 points) Use linear approximation to estimate $\sqrt[7]{129}$, by linearizing the function $f(x) = \sqrt[7]{x}$ at a suitable value. (HINT: What number near 129 do we know the 7th root of? NOTE: This problem should not require a calculator. Provide your answer as a fraction.)

Let $f(x) = x^{1/7}$ $\quad f(128) = 2$

$f'(x) = \frac{1}{7} x^{-6/7}$ $\quad f'(128) = \frac{1}{7} \cdot \frac{1}{(\sqrt[7]{x})^6} = \frac{1}{7 \cdot 64} = \frac{1}{448}$



$y = \frac{1}{448}(x - 128) + 2$

↑ tangent line $L(x)$

$L(129) = 2 + \frac{1}{448} = \frac{897}{448} \approx 2.0022$

4. (10 points) Let $f(x) = 2x^3 - 6x + 5$. Find the maximum and minimum values of f on the interval $-2.5 \leq x \leq 1.5$.

Find Critical Points

$f'(x) = 6x^2 - 6$

Set $= 0$

$x^2 = 1$

$x = \pm 1$

+5

Candidates		f
ends	-2.5	-11.25
	1.5	2.75
cps	-1	
	$+1$	

$f(-2.5) = -31.25 + 15 + 5 = -11.25$ ← Min

$f(1.5) = 3.75 - 9 + 5 = 2.75$

$f(-1) = -2 + 6 + 5 = 9$ ← Max

$f(1) = 2 - 6 + 5 = 1$

+5

Local data?

Bonus +1

5. (15 points) Find all the vertical and horizontal tangent lines of the curve given by the equation:

$$y^2 = x^3 - 12x.$$

5 pts

$2y \cdot y' = 3x^2 - 12$

$y' = \dfrac{3x^2 - 12}{2y}$

H.A. top = 0

5 pts $3x^2 - 12 = 0$ $x^2 = 4$ $x = \pm 2$

$y = 4$

$y = -4$

2 for these

y?

$y^2 = 8 - 24 = -16$ none

$y^2 = -8 + 24 = 16$

$y = \pm 4$ 3 more

VA $2y = 0$

5 pts $y = 0$

$x^3 - 12x = 0$

$x(x^2 - 12) = 0$

$x = 0$

$x = \pm\sqrt{12}$