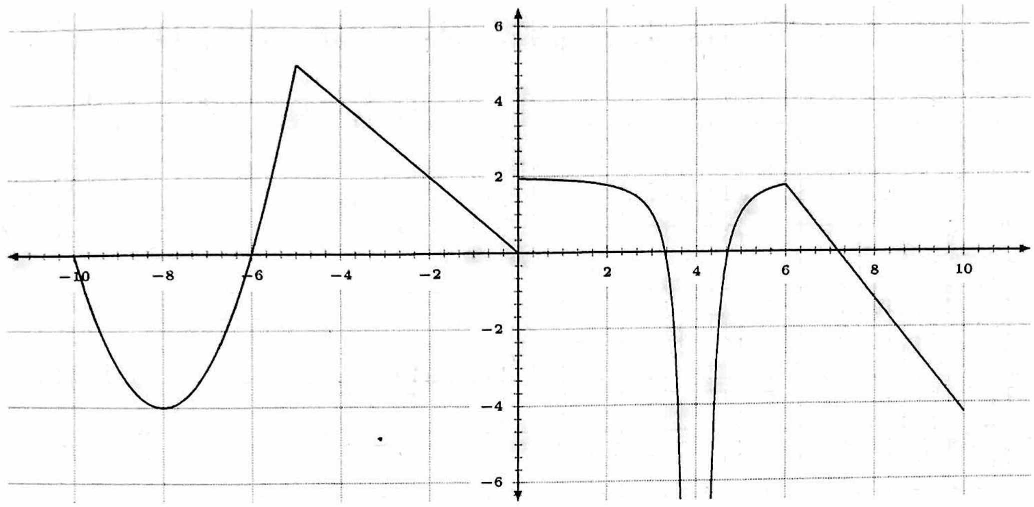
1. The following is a graph of the function $f(x)$.



(a) (8 points) Compute the following limits if they exist. If they do not exist, explain why.

Note $\lim \neq \frac{d}{dx}$

$\lim_{x\to -8} f(x)$	$\lim_{x\to -5} f(x)$	$\lim_{x\to 0} f(x)$	$\lim_{x\to 4} f(x)$
-4	5	DNE	$-\infty$ (1 if DNE)

(b) (2 points) Find all x values where $f'(x) = 0$.

$x = -8$ (+1) (+1 for no extras)

(c) (3 points) On which intervals is $f'(x) > 0$?

$-8 < x < -5$, $4 < x < 6$

(d) (3 points) At which x coordinates is f continuous but *not* differentiable?

$x = -5$ (+1) $x = 6$ (+1) (+1 no extras)

(e) (2 points) Give the equations of any vertical asymptotes.

$x = 4$

(f) (3 points) Compute the following limit.

$$\lim_{x\to -2} \frac{f(x) - f(-2)}{x+2}. = f'(-2) = -1$$

(g) (3 points) What is $f''(8)$?

0

2. A coin is dropped from the top of the Empire State Building, from a height of 400m. It's height t seconds after being dropped is given by the equation $h(t) = 400 - 4.9t^2$.

(a) (3 points) Find the velocity after it has been falling for 2 seconds.

$h'(t) = -9.8t$

$t'(2) = -9.8 \cdot 2 = -19.6 \frac{m}{s}$

-1/2 for sign errors

(b) (3 points) When is the coin falling at 50 m/s?

$-9.8t = -50$

$t = \frac{-50}{-9.8} \approx 5.102$ sec.

(c) (3 points) Find the acceleration of the coin in m/s^2.

$a = h''(t) = -9.8\, m/s^2$

(d) (6 points) What is the velocity of the coin when it hits the ground?

1 When does $h(t) = 0$?

$4.9t^2 = 400$

$t^2 = 81.63$

$t = 9.04$ seconds

3 points

2 $v(9.04) = -9.8(9.04)$

$\boxed{= 88.59\, m/s.}$

$x(t) = t - \sin t$

③ $y(t) = 1 - \cos t$

(a)

Pt $x(\pi/3) = \pi/3 - \sin(\pi/3) = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$

$y(\pi/3) = 1 - \cos \pi/3 = 1 - 1/2 = 1/2$

Slope

$x'(t) = 1 - \cos t$ $\quad x'(\pi/3) = 1 - 1/2 = 1/2$

$y'(t) = \sin t$ $\quad y'(\pi/3) = \sqrt{3}/2$

$m = y'/x' = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

Line $\quad y = \sqrt{3}\left(x - \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) + \frac{1}{2}$

No k? Max 4

(b)

Note $\quad x'(t) = 0$

$\cos t = 1 \implies t = 0 + 2\pi \cdot k$

$y'(t) = 0$

$\hookrightarrow \sin t = 0 \implies t = 0 + \pi k.$

Therefore $\quad$ Horizontal @ $t = \pi + 2\pi k$

Vertical?

$$\lim_{\substack{x \to 0 \\ t = 0 + 2\pi \cdot k}} \frac{\sin t}{1 - \cos t} \cdot \frac{1 + \cos t}{1 + \cos t} = \frac{\sin t + \sin t \cos t}{\sin^2 t} \neq \frac{\sin t}{\sin^2 t} + \frac{\sin t \cos t}{\sin^2 t} = \frac{1 + \cos t}{\sin t} \to \infty$$

DO! Plugging in #s? Max = 1

4. Compute the following limits. Indicate to me which one you skip. Show and justify all steps! If the limit does not exist explain why.

(a) (5 points)

$$\lim_{x\to\infty} \sqrt{16x^2 - 24x} - 4x \cdot \frac{\sqrt{16x^2-24x}+4x}{\sqrt{16x^2-24x}+4x}$$

$$= \lim_{x\to\infty} \frac{16x^2-24x-16x^2}{\sqrt{16x^2-24x}+4x} = \lim_{x\to\infty} \frac{-24x}{\sqrt{16x^2-24x}+4x}$$

$$= \lim_{x\to\infty} \frac{-24}{\sqrt{16-24/x}+4} = \frac{-24}{4+4} = -3$$

(b) (5 points) (Hint: Compute the limit from each side)

$$\lim_{x\to -1} \frac{|x+1|}{x+1}.$$

$$\lim_{x\to -1^-} \frac{|x+1|}{x+1} = \lim_{x\to -1^-} \frac{-(x+1)}{x+1} = \lim_{x\to -1^-} -1 = -1$$

$$\lim_{x\to -1^+} \frac{|x+1|}{x+1} = \lim_{x\to -1^+} \frac{x+1}{x+1} = \lim_{x\to -1^+} +1 = +1$$

$1 \neq -1$

So DNE

(c) (5 points)

$$\lim_{z\to 0} \frac{(2+z)^8 - 2^8}{z}. = f'(2) \text{ where } f(x) = x^8$$

Thus $f'(x) = 8x^7$

$f'(2) = 8 \cdot 2^7 = 2^{10} = 1024$

Bonus (+1) to those who noticed this

L'Hôpital Max = 1

5. Compute the $\frac{dy}{dx}$ for 5 of the following curves. Indicate to me which one you skip. Show and justify all steps! Once computed, you do not need to simplify the derivative.

(a) (5 points)

$$f(x) = \sqrt[10]{x^5 + x^4 + x^3 + x^2 + x + 1}. = (x^5+x^4+x^3+x^2+x+1)$$

$$f'(x) = \frac{1}{10}(x^5+x^4+x^3+x^2+x+1)^{-9/10} \cdot (5x^4+4x^3+3x^2+2x+1)$$

(b) (5 points)

$$x^3 - e^{xy} + \sin(y) = 13.$$

Implicit:

$$3x^2 - e^{xy}y - e^{xy}xy' + \cos y \cdot y' = 0$$

$$y'(\cos y - e^{xy}x) = e^{xy}y - 3x^2$$

$$y' = \frac{e^{xy}y - 3x^2}{\cos y - e^{xy}x}$$

(c) (5 points)

Product

$$y = \arctan(x)\ln(1/x^2).$$

$$y' = \frac{1}{1+x^2}\ln(1/x) + \arctan x\left(\frac{1}{1/x^2} \cdot \frac{-2}{x^3}\right)$$

+2

+3

miss chain?

-1.5

(d) (5 points)

CHAIN

$$h(x) = 3^{\ln(\sin(x^2))}.$$

$h'(x) = \ln 3 \cdot 3^{\ln(\sin(x^2))} \cdot \left(\frac{1}{\sin(x^2)}\right) \cdot (\cos x^2) \cdot (2x)$

No Chain?

-4

(e) (5 points)

$$l(t) = \frac{3x^2\sqrt{2x-5}}{\sin(x)\cos(x)}.$$

Log Diff

$\ln(y) = \ln(3x^2\sqrt{2x-5}) - \ln(\sin x \cos x)$

$= \ln 3 + 2\ln(x) + \frac{1}{2}\ln(2x-5)$

$- \ln \sin x - \ln \cos x$

$\frac{y'}{y} = \frac{2}{x} + \frac{1}{2x-5} - \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ | $y' = y(\text{this})$.

(f) (5 points)

$$y = \sin(x)^{\cos(x)}.$$

log diff

$\ln y = \cos x \ln \sin x$

$\frac{y'}{y} = -\sin x \ln \sin x + \cos x \cdot \frac{\cos}{\sin x}$

$y' = \sin x^{\cos x}\left(\frac{\cos^2 x}{\sin x} - \sin x \ln \sin x\right).$

No log? No credit