Math 120 A Midterm 2 (60 minutes) July 31, 2018

Name: Key

Directions:

- You have 60 minutes to complete this exam.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 20 | |
| 4 | 15 | |
| Total: | 60 | |

1. (a) (5 points) Solve for $x$.

$$2^{x^2} = 8^{x+6}.$$

Apply ln $\quad x^2 \ln(2) = (x+6)(\ln(8))$

$$x^2 = (x+6)\left(\frac{\ln(8)}{\ln(2)}\right) \leftarrow = 3$$

$$x^2 = 3x + 18$$

$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$x = 6$$

or

$$x = -3$$

(b) (5 points) Graph the following function. Label both asymptotes and the point you use to determine orientation.

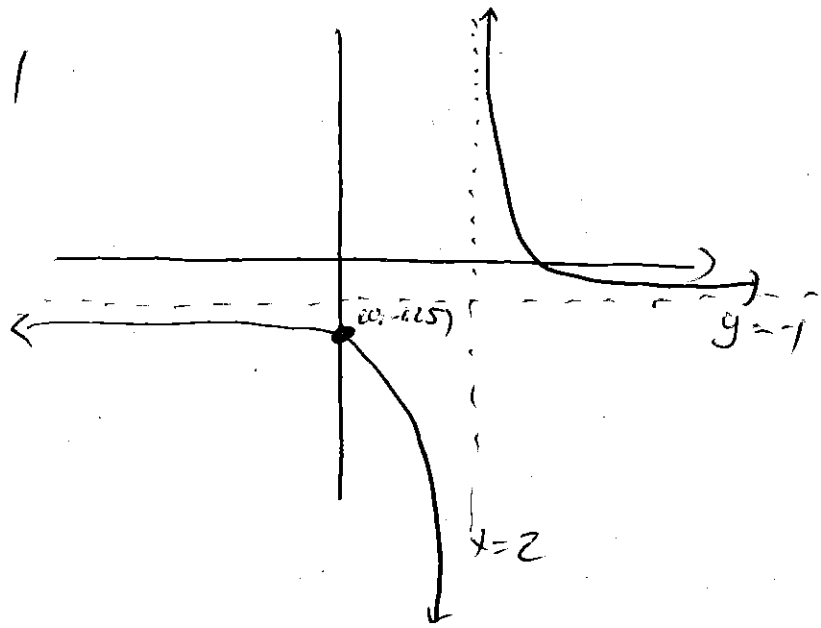
$$f(x) = \frac{-2x+5}{2x-4}.$$

V.A. $x = \frac{-d}{c} = \frac{4}{2} = 2$

H.A. $y = \frac{a}{c} = \frac{-2}{2} = -1$

Pt $f(0) = \frac{-5}{4}$

$= -1.25$



2. A time-destroying dictator has ordered the destruction of every watch, clock, calendar and sundial. Using your quick wit, you devise a clever device to keep track of time. Your invention involved setting aside 2000g of Temporium, a mildly radioactive compound with a half life of 112 days, in a controlled environment.

(a) (5 points) Write a function $W(t)$, which returns the weight of the Temporium after $t$ days. Carefully state the domain and range.

$W(0) = A_0 b^0 = 2000$

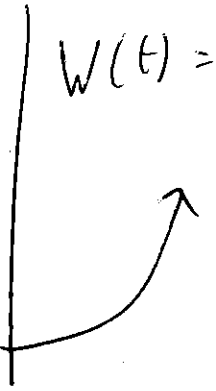
$\Rightarrow A_0 = 2000$

$W(1) = 2000 \cdot b^{112} = 1000$

$\Rightarrow b^{112} = 1/2$

$b = \sqrt[112]{\frac{1}{2}} = \left(\frac{1}{2}\right)^{\frac{1}{112}}$

$W(t) = 2000\left(\frac{1}{2}\right)^{t/112}$



Domain: $t \geq 0$

Range: $0 < W(t) \leq 2000$

(b) (5 points) Finish your radioactive clock by inverting the function from part (a). Explain in words what this function does (i.e., what is the input? What is the output?). As usual, carefully state the domain and range.

describes amount of time passed given a weight of remaining Temporium.

$W = 2000\left(\frac{1}{2}\right)^{t/112}$

$\left(\frac{1}{2}\right)^{t/112} = \frac{W}{2000}$

Apply ln

$\frac{t}{112}\ln\left(\frac{1}{2}\right) = \ln\left(\frac{W}{2000}\right)$

$t = 112\,\frac{\ln(W/2000)}{\ln(1/2)}$

Domain: $0 < W(t) \leq 2000$

Range: $t \geq 0$

(c) (5 points) Rumors of your secret clock land you in jail, but it was never discovered. You escape after what feels like ages, and go to your clock to discover that 50mg of Temporium remain. How long has it been? (NOTE: 1g = 1000mg)

$50\,mg = .05g$

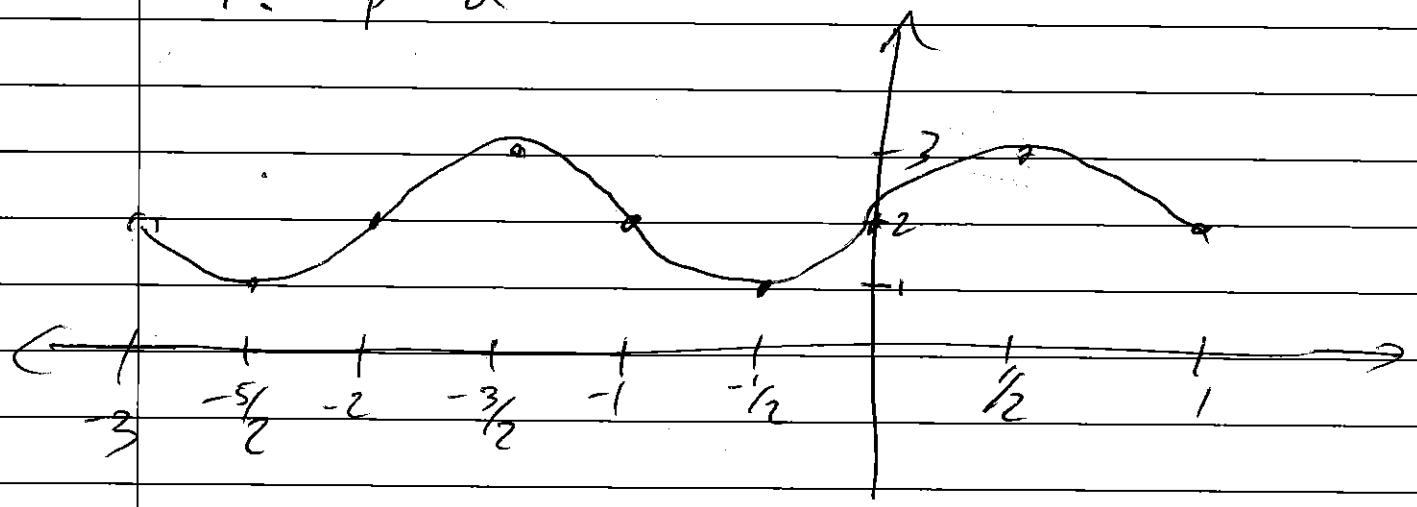
$t(W) = t(.05) = 112\,\frac{\ln(.05/2000)}{\ln(1/2)}$

$= 1712.22$ days.

(3)

(a)

1. Compress horiz by $\pi$
2. Left 1
3. Reflect over x-axis
4. Up 2




D: $-3 \le x \le -1$ R: $1 \le y \le 3$

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(b)

$a = -2$

$b = 3$

$c = 2$

$d = 0$

Domain

$-2\pi \le \frac{1}{2}x \le 2\pi$

$-4\pi \le x \le 4\pi$

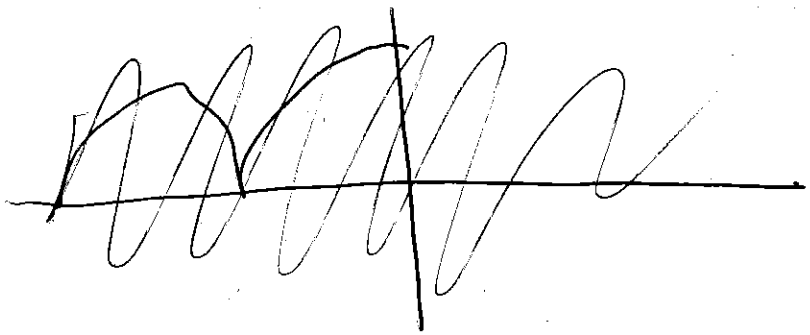
Range

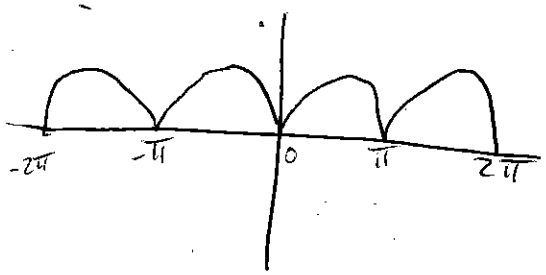
$-1 \le y \le 1$

$-3 \le 3y \le 3$

$-5 \le 3y - 2 \le 1$

(c) (5 points) Graph $|f(x)|$, and write its multipart rule. What is the domain and range?






$$|f(x)| = \begin{cases} \sin x & -2\pi \le x \le -\pi \\ & 0 \le x \le \pi \\ -\sin x & -\pi \le x \le 0 \\ & \pi \le x \le 2\pi \end{cases}$$

D: $-2\pi \le x \le 2\pi$

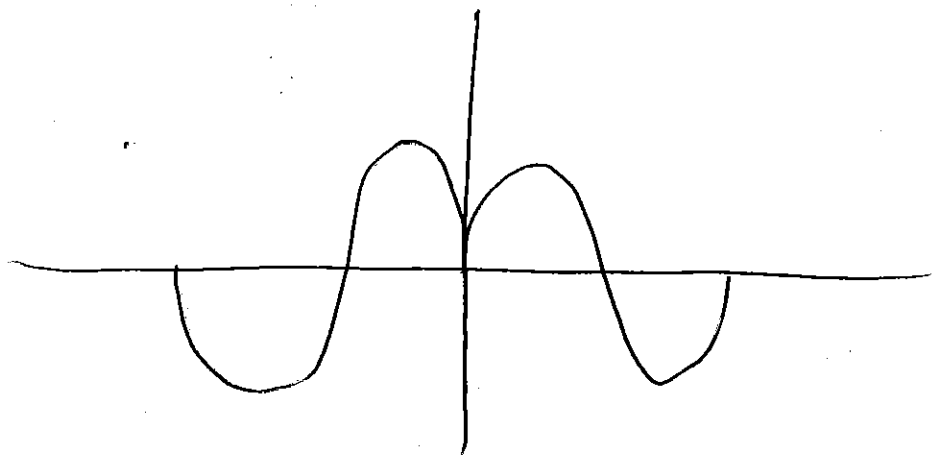
R: $0 \le y \le 1$

(d) (5 points) Graph $f(|x|)$ and write its multipart rule. What is the domain and range?

$$f(|x|) = \begin{cases} \sin x & 0 \le x \le 2\pi \\ \sin(-x) & -2\pi \le x \le 0 \end{cases}$$

D: $-2\pi \le x \le 2\pi$

R: $-1 \le y \le 1$



## Problem 4

(a) $v(t) = \dfrac{at+b}{t+d} = \dfrac{3\times10^8 \cdot t + b}{t+d}$

H.A $\Rightarrow$ $a = 3\times10^8$

① $(0, 100) \Rightarrow \dfrac{b}{d} = 100 \Rightarrow b = 100d$

② $(2, 100{,}000) \Rightarrow \dfrac{6\times10^8 + b}{2+d} = 100\,000$

$\Rightarrow 6\times10^8 + b = 200\,000 + 100\,000\,d$

Plug in ①

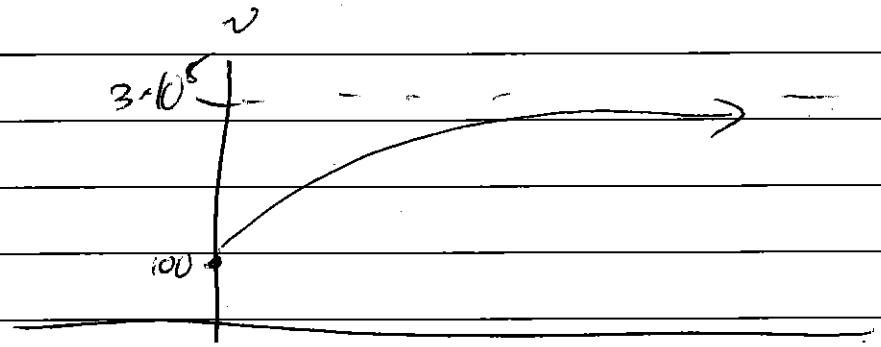
$6\times10^8 + 100d = 200\,000 + 100\,000\,d$

$99900d = 6\times10^8 - 200000$

$d = \dfrac{6\times10^8 - 200000}{99900} \approx 6004.004004$

$b = 100d = 600400.4004\ldots$

$v(t) = \dfrac{3\times10^8 t + 600400.4004}{t + 6004.004}$ D: $t \geq 0$



(b) $v^{-1}(t) = \dfrac{-600t^2 + 600t + 4}{t - 3 \times 10^8}$

D: $100 \leq v < 3 \times 10^8$

R: $t \geq 0$

(c)

A) $v^{-1}(9.8 \cdot 10^5) = 19.6754$ mins

B) $v^{-1}(3.6 \cdot 10^9) = 72.9211$ mins

$72.9211 - 19.6754 = 53.2457$

B first

A 53.2457 mins later