Math 120 A Midterm 1 July 12, 2018

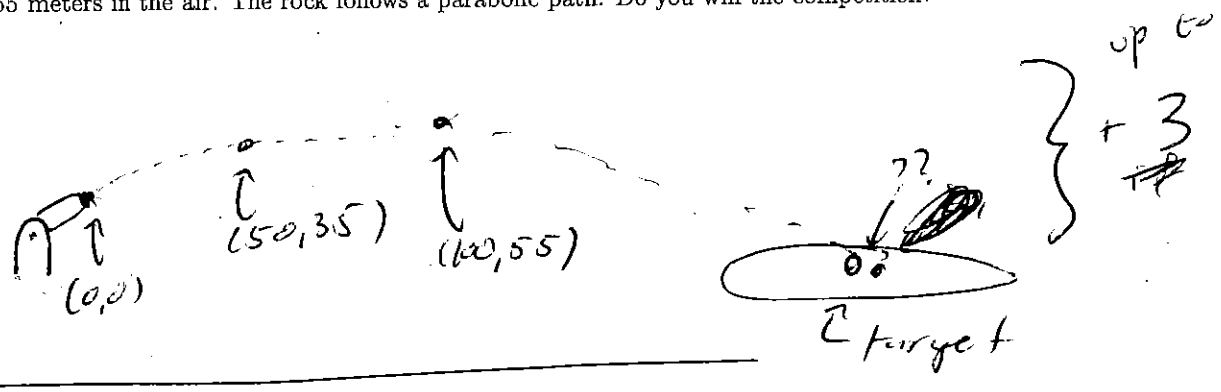
Name: Key

Directions:

- You have 60 minutes to complete this exam.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet of notes on regular 8.5-11 paper. Two sided is ok.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you need extra room, use the back side of the page. Include a note to indicate to the reader that you have done so.
- If you have any questions, raise your hand.

Question	Points	Score
1	10	
2	10	
3	20	
4	10	
Total:	50	

1. (10 points) To win a contest you must use a cannon to launch a cannonball within 10 meters of a target located 275 meters away. 50 meters out, the cannonball is 35 meters in the air, and 100 meters out it is 55 meters in the air. The rock follows a parabolic path. Do you win the competition?



$f(x) = ax^2 + bx + c$

i) $0 = a\cdot 0^2 + b\cdot 0 + c$ ← $c = 0$

ii) $35 = a\cdot 50^2 + b\cdot 50 + c$ ⟹ $35 = 2500a + 50b$

iii) $55 = a\cdot 100^2 + b\cdot 100 + c$ ⟹ $55 = 10000a + 100b$ } +2

(iii) -2(ii): $-15 = 5000a$

$\rightarrow a = \frac{-15}{5000} = \frac{-3}{1000}$

Plug into (iii)

$55 = 10000\left(\frac{-3}{1000}\right) + 100b$

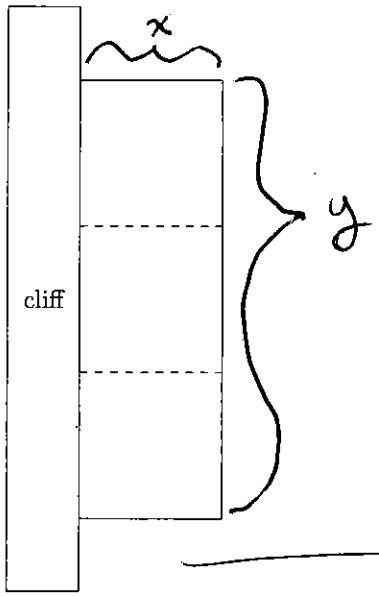
$85 = 100b \quad b = \frac{85}{100} = \frac{17}{20}$ } +2

$f(x) = \frac{-3}{1000}x^2 + \frac{17}{20}x = x\left(\frac{-3}{1000}x + \frac{17}{20}\right) = 0$

$\hookrightarrow x = \frac{-17/20}{-3/1000} \approx 283.33$

within 10 of 275 so we win! } +3

2. (10 points) A farmer wishes to build a 3 chambered enclosure alongside a cliff. The exterior fencing costs 35 dollars per meter, while the interior fencing separating the chambers costs 20 dollars per meter. If the farmer has 4500 dollars to spend, what is the maximum area they can enclose. (NOTE: There does not need to be any fencing along the cliff).



$\text{Area} = xy$

$\text{Cost} = 35x + 20x + 20x + 35x + 35y$

$\hookrightarrow 110x + 35y = 4500$

$y = \dfrac{4500 - 110x}{35}$

$\text{Area} = x\left(\dfrac{4500 - 110x}{35}\right)$

$= -\dfrac{110}{35}x^2 + \dfrac{4500}{35}x$

$= -\dfrac{22}{7}x^2 + \dfrac{900}{7}x$

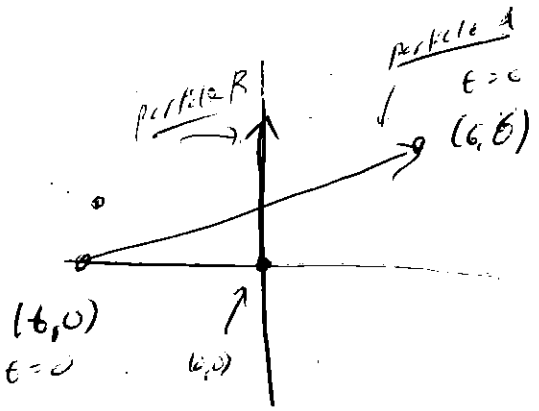
Max @ 20.455

$-b/2a = \dfrac{-900/7}{-(22/7)2} = \dfrac{450}{22} \approx$ [illegible]

$f(20.455) = 1314.94$

3. The following problem has a fixed coordinate plane ruled in millimeters. Particle A travels in a straight line from coordinates $(-6, 0)$ towards the point $(6, 6)$, reaching it in 6 seconds. At the same time, a second particle (call it particle B), leaves from the origin and moves straight up along the y-axis traveling at 5 millimeters per second.

(a) (5 points) Write parametric equations for each particle.



Particle A

$x_A(t) = \frac{6-(-6)}{6-0}(t-0) + 6$

$= 2t - 6$

$y_A(t) = \frac{6-0}{6-0}(t-0) + 0 = t$

Particle B

$x_B(t) = 0$

$y_B(t) = 5t$

(b) (5 points) Write a function $d(t)$, where t is time, in seconds, and $d(t)$ is distance between the two particles, in millimeters.

$d(t) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$= \sqrt{(2t-6)^2 + (t-5t)^2}$

$= \sqrt{4t^2 - 24t + 36 + 16t^2}$

$= \sqrt{20t^2 - 24t + 36}$

(c) (5 points) What is the minimum distance between the two particles?

$$d(t) = \sqrt{20t^2 - 24t + 36}$$

min @ $\frac{-b}{2a} = \frac{24}{40} = \frac{3}{5}$ seconds

$$d\left(\frac{3}{5}\right) = \sqrt{20 \cdot \frac{9}{25} - 24\left(\frac{3}{5}\right) + 36}$$

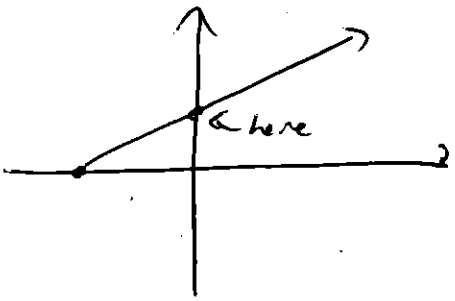
$$= \sqrt{\frac{36}{5} - \frac{72}{5} + \frac{180}{5}}$$

$$= \sqrt{\frac{144}{5}} \approx 5.37 \text{ mm}$$

Don't plug into D?
-2

Don't take $\sqrt{}$
-1

(d) (5 points) The two paths cross at some point. Which point is this?



When $x_A(t) = 0$

$2t - 6 = 0$

$t = 3$

$y_A(3) = 3$

$x_A(3) = 0$

So (3, 0)

4. Let $f(x)$ be given by the following multipart rule.

$$f(x) = \begin{cases} 2x+5 & x \leq 0 \\ x^2 - 2x + 5 & x \geq 0 \end{cases}$$

(a) (5 points) Sketch a graph of $y = f(x)$. (HINT: You'll need to put the second part in vertex form to graph it).

$x^2 - 2x + 5$

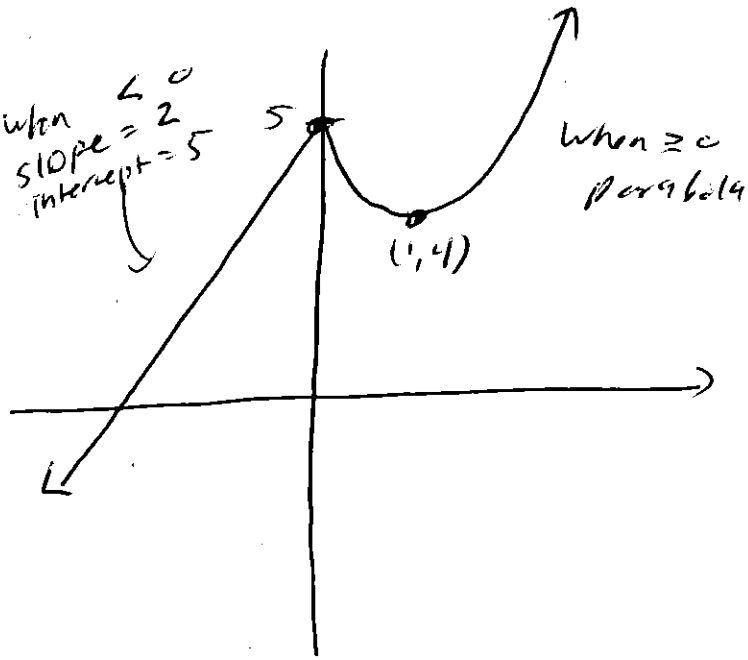
vertex

$h = -b/2a = \frac{2}{2} = 1$

$f(1) = 1 - 2 + 5$

$= 4$

$(1, 4)$



(b) (5 points) Find all x such that $f(x) = 8$.

$8 = 2x + 5 \Rightarrow x = \frac{3}{2} > 0$ so not in domain (or use graph).

I would say...

$8 = x^2 - 2x + 5 \Rightarrow x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$

or

$x = -1$ ← not in domain

only this

Checking in D is 3 pts