Math 120 A Midterm 1 (60 minutes) July 13, 2017

Name: Key

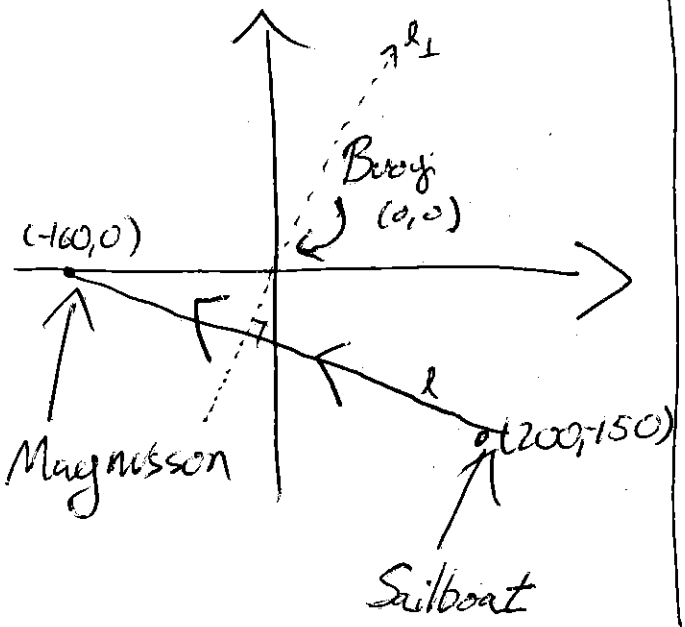
Directions:

- You have 60 minutes to complete this exam.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

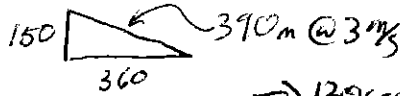
Question	Points	Score
1	15	
2	20	
3	10	
4	10	
Total:	55	

1

(a)



(b)



$x(t) = \frac{\Delta x}{\Delta t} t + x_0$

$= \frac{-360}{130} t + 200$

$= -2.769t + 200$ ← (or +360)

$y(t) = \frac{\Delta y}{\Delta t} t + y_0$

$= \frac{+150}{130} t - 150$

$= +1.154t - 150$

(c) ℓ: (200, -150)
(-160, 0)

$y = \frac{-5}{12}(x + 160)$

$= \frac{-5}{12}x - \frac{200}{3}$

$\ell_\perp$: slope: $\frac{12}{5}$

intercept: 0

$y = \frac{12}{5}x$

Intersect

$\frac{12}{5}x = \frac{-5}{12}(x + 160)$

$\Rightarrow x = \frac{-4000}{164} = -23.67$

$y = \frac{12}{5}x = -56.80$

(-23.67, -56.80)

(d) $d(t) = \sqrt{x(t)^2 + y(t)^2}$

$= \sqrt{9t^2 - \frac{18400}{13}t + 62500}$

Min @ $-b/2a$

$= \frac{1050}{13}$

≈ 80.769

$x\left(\frac{1050}{13}\right) = \frac{-4000}{169}$ ✓

$y\left(\frac{1050}{13}\right) = \frac{-9600}{169}$ ✓

2. A parabola contains the three points, $(0,-12)$, $(2,4)$, and $(5,-2)$.

(a) (5 points) Write the equation of the parabola in standard form $f(x) = ax^2 + bx + c$.

System

i) $a\cdot 0^2 + b\cdot 0 + c = -12 \} c = -12$

ii) $a(2)^2 + b(2) + c = 4$

iii) $a\cdot 5^2 + b\cdot 5 + c = -2$

$\rightarrow$ (i) $4a + 2b = 16 \rightsquigarrow b = 8 - 2a$

(ii) $25a + 5b = 10$

$25a + 5(8-2a) = 10$

$15a = -30$

$a = -2$

$b = 8 - 2(-2)$

$= 8 + 4 = 12$

$f(x) = -2x^2 + 12x - 12$

(b) (5 points) Convert $f(x)$ into vertex form $f(x) = a(x-h)^2 + k$ and then sketch a graph of $y = f(x)$, labeling the vertex.

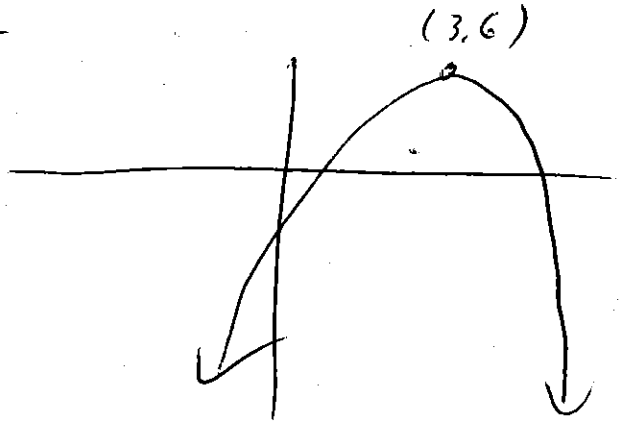
$h = \frac{-b}{2a} = \frac{-12}{-4} = 3$

$k = f(3) = -2\cdot 9 + 12\cdot 3 - 12$

$= -18 + 36 - 12$

$= 6$

$y = -2(x-3)^2 + 6$.



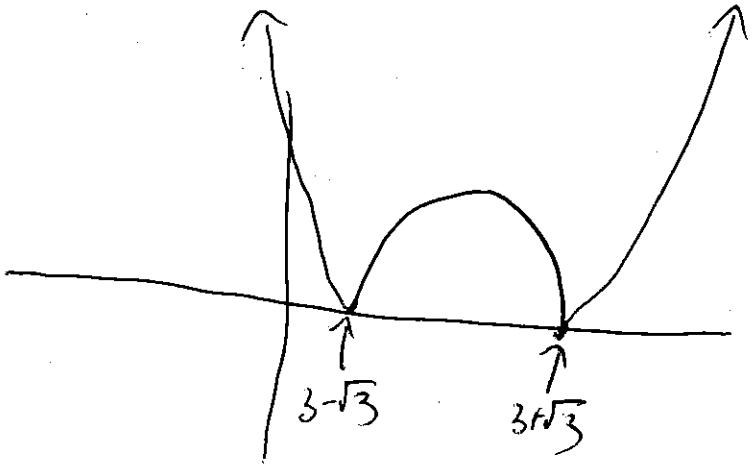
~~Find x-intercepts of~~

(c) (5 points) Sketch $y = |f(x)|$, and write the associated multipart rule.

$-2(x-3)^2+6 = 0$

$(x-3)^2 = 3$

$x = 3 \pm \sqrt{3}$



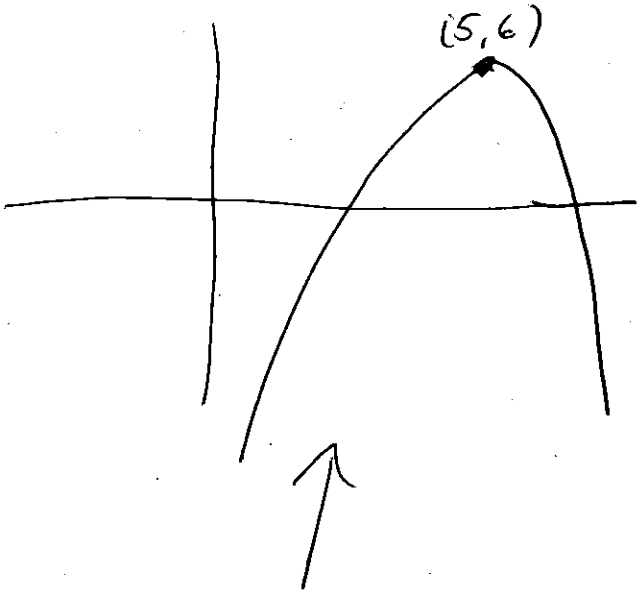
$$|f(x)| = \begin{cases} 2x^2-12x+12 & x \le 3-\sqrt{3} \\ -2x^2+12x-12 & 3-\sqrt{3} \le x \le 3+\sqrt{3} \\ 2x^2-12x+12 & x \ge 3+\sqrt{3} \end{cases}$$

$1.268 \downarrow x \le 3-\sqrt{3}$

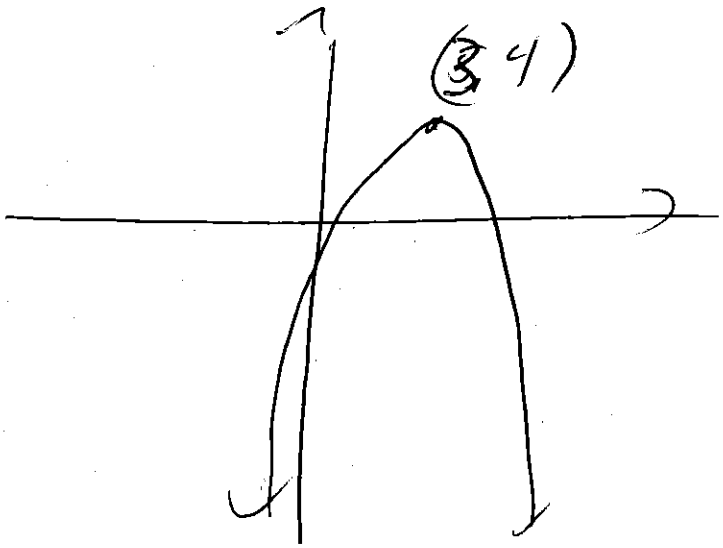
$3-\sqrt{3} \le x \le 3+\sqrt{3}$

$x \ge 3+\sqrt{3} \uparrow 1.732$

(d) (5 points) Let $g(x) = x - 2$. Sketch the compositions $f \circ g(x)$ and $g \circ f(x)$, labeling the coordinates of the vertex for each. (HINT: Can you express what is happening in terms of shifts?).

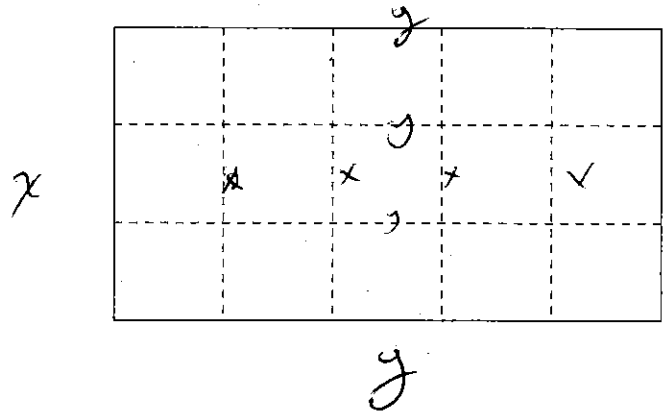


Shift right 2



shift down 2

3. (10 points) You would like to build a rectangular enclosure partitioned into a grid, as pictured below. The exterior fencing (solid lines) costs \$25 a foot, and the fencing for the interior partitions (dashed lines) cost \$12 a foot. If you have \$11500 to spend, what is the maximum area you can enclose?



$25(2y) + 25(2x) + 12(2y) + 12(4x) = 11500$

$74y + 98x = 11500$

$y = \frac{11500 - 98x}{74}$

Maximize

$A = xy$

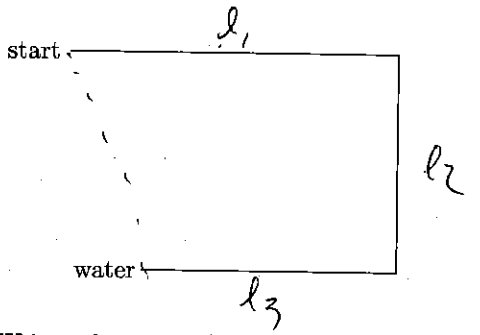
$= x\left(\frac{11500 - 98x}{74}\right)$

$= \frac{11500}{74}x - \frac{98}{74}x^2$

Max @ $x = \frac{-5}{2a} = \frac{11500}{2 \cdot 98} = \frac{2875}{49} = 58.67$

$A(x) = 4559.087$

4. (10 points) Oscar wanders aimlessly through through the Utah desert. He first walks East at 3 mph for 3 hours. He then walks south at 4 mph for 1.5 hours. Finally he walks west at 2 mph until he reaches a water source 6 miles south and 2 miles east of his starting point. There he stops. His path should like this:



Write a function $d(t)$ for his distance (in miles) after t hours of wandering. This should be a multipart function, and you should assume Oscar stops moving once he reaches the water source.

ℓ_1:

$d(t) = 3t$ (+2)

ℓ_2 | Parametrics

$x(t) = 9$

$y(t)$ @ 2 points

$t = 3$ $y = 0$

$t = 4.5$ $y = -6$

$y(t) = \frac{-6}{1.5}t + \cancel{0}$ (+3)

$= -4(t - 3)$

ℓ_3

$y(t) = -6$

$x(t) = -2(t - 4.5) + 9$

$x(t) = 2$

$-2(t - 4.5) + 9 = 2$

$-2t + 18 = 2$

$-2t = -16$ (+3)

$t = 8$

$$d(t) = \begin{cases} 3t & 0 \le t \le 3 \\ \sqrt{9^2 + (-4(t-3))^2} & 3 \le t \le 4.5 \\ \sqrt{(-2t+18)^2 + (6)^2} & 4.5 \le t \le 8 \\ \sqrt{2^2 + 6^2} = \sqrt{40} & t \ge 8 \end{cases}$$

(+1)

1 free