

Name: Key

Answer the questions in the spaces provided. Show all necessary work. If you run out of space, use the back side and leave a message to indicate that you have done so. If you have any questions, raise your hand and I will come try to answer.

1. In a scientific marvel, engineers have built a small self replicating robot. One of these robots can create ten copies of itself in a day! A robot will not self destruct after a days work, and will instead stay on to keep building. Therefore the number of robots grows at an exponential rate.

(a) (8 points) Write a function $R(t)$ for the number of robots there are after t days.

$$R(t) = R_0 \cdot b^t$$

$$R(0) = 1 = R_0$$

$$R(1) = 11 = R_0 \cdot b$$

$$\Rightarrow b = 11$$

$$\text{So } R(t) = 11^t \quad \text{If } \frac{10^t}{-2}$$

(b) (4 points) The robots are relatively small, covering only 25mm^2 of ground space. Write $A(t)$, a function for the total area covered by robots after t days.

$$R(t) = 25 \cdot 11^t$$

(c) (8 points) The earth has a total surface area of 510.1 million km^2 . How long does it take for the robots to cover the entire surface of the globe. (KEEP AN EYE ON UNITS!)

$$510.1 \times 10^6 \text{ km}^2$$

$$= 5.101 \times 10^8 \text{ km}^2 = 5.101 \times 10^{20} \text{ km}^2$$

$$1 \text{ km}^2 = 10^{12} \text{ mm}^2$$

$$1 \text{ km}^2 \left(\frac{10^6 \text{ mm}}{1 \text{ km}} \right) \left(\frac{10^6 \text{ mm}}{1 \text{ km}} \right)$$

3 points

$$\text{million} = 10^6 \leftarrow 1 \text{ pt}$$

$$10^6 \text{ mm}^2 = 1 \text{ km}^2 \leftarrow 2 \text{ pts}$$

Instead
 $10^6 \leftarrow -1$
 else -2

$$25 \cdot 11^t = 5.101 \cdot 10^{20}$$

$$11^t = \frac{5.101 \cdot 10^{20}}{25}$$

$$t \cdot \ln(11) = \ln\left(\frac{5.101 \cdot 10^{20}}{25}\right)$$

$$t = \frac{\ln\left(\frac{5.101 \cdot 10^{20}}{25}\right)}{\ln(11)}$$

$$\approx 18.5 \text{ days}$$