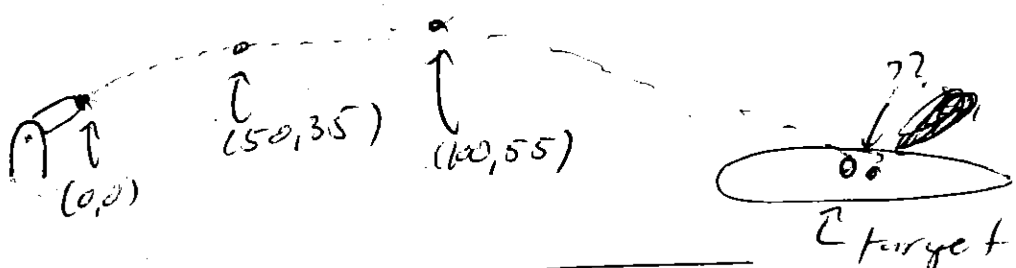
1. (10 points) To win a contest you must use a cannon to launch a cannonball within 10 meters of a target located 275 meters away. 50 meters out, the cannonball is 35 meters in the air, and 100 meters out it is 55 meters in the air. The rock follows a parabolic path. Do you win the competition?

up to +3



$f(x) = ax^2 + bx + c$

i) $0 = a\cdot 0^2 + b\cdot 0 + c \leftarrow c = 0$

ii) $35 = a\cdot 50^2 + b\cdot 50 + c \implies 35 = 2500a + 50b$

iii) $55 = a\cdot 100^2 + b\cdot 100 + c \qquad 55 = 10000a + 100b$ +2

(iii) − 2(ii): $-15 = 5000a$

$\longrightarrow a = \frac{-15}{5000} = \frac{-3}{1000}$

Plug into (iii)

$55 = 10\cancel{000}\left(\frac{-3}{\cancel{1000}}\right) + 100b$

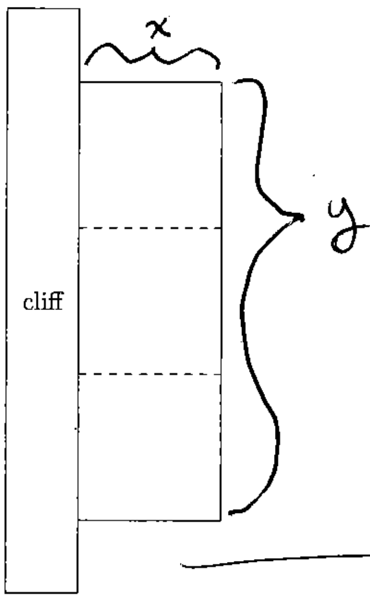
$85 = 100b \qquad b = \frac{85}{100} = \frac{17}{20}$ +2

$f(x) = \frac{-3}{1000}x^2 + \frac{17}{20}x = x\left(\frac{-3}{1000}x + \frac{17}{20}\right) = 0$

$\hookrightarrow x = \frac{-17/20}{-3/1000} \approx 283.33$

within 10 of 275 so we win! +3

2. (10 points) A farmer wishes to build a 3 chambered enclosure alongside a cliff. The exterior fencing costs 35 dollars per meter, while the interior fencing separating the chambers costs 20 dollars per meter. If the farmer has 4500 dollars to spend, what is the maximum area they can enclose. (NOTE: There does not need to be any fencing along the cliff).



$$\text{Area} = xy$$

$$\text{Cost} = 35x + 20x + 20x + 35x + 35y$$

$$110x + 35y = 4500$$

$$y = \frac{4500 - 110x}{35}$$

$$\text{Area} = x\left(\frac{4500 - 110x}{35}\right)$$

$$= -\frac{110}{35}x^2 + \frac{4500}{35}x$$

$$= -\frac{22}{7}x^2 + \frac{900}{7}x$$

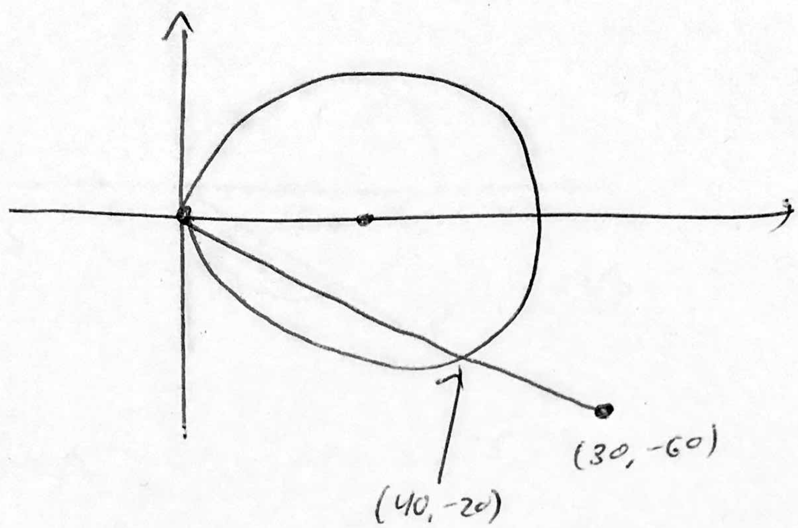
Max @

$$-b/2a = \frac{-900/7}{-(22/7)2} = \frac{450}{22} \approx 20.455$$

$$f(20.455) = 1314.94$$

3. A streetlamp illuminates a radius of 25 meters, and is located 25 meters east of the door to the library. A student is 60 meters east and 30 meters south of the library door. They walk in a straight line to the library door, and arrive in 90 seconds.

(a) (5 points) Draw a picture of the situation. Choose coordinates, and label the streetlamp, the circle of illumination, the library door, the students starting position, and their path.



(b) (5 points) How much time does the student spend illuminated?

Student $y = -\frac{1}{2}x$

Circle $(x-25)^2 + y^2 = 25^2$

Plug in $y = \frac{-1}{2}x$

$(x-25)^2 + \frac{x^2}{4} = 25^2$

{ solve

$1.25x^2 - 50x = 0$

$x(1.25x - 50) = 0$

$x = 0$

or

$1.25x = 50$

$x = 40$

$y = -20$

distance illuminated

$\sqrt{40^2 + 20^2}$

$= \sqrt{1600 + 400}$

$= \sqrt{2000}$

Rate? $\sqrt{30^2 + 60^2}$

$= \sqrt{900 + 3600}$

$= \sqrt{4500}$

$d = r \cdot t$

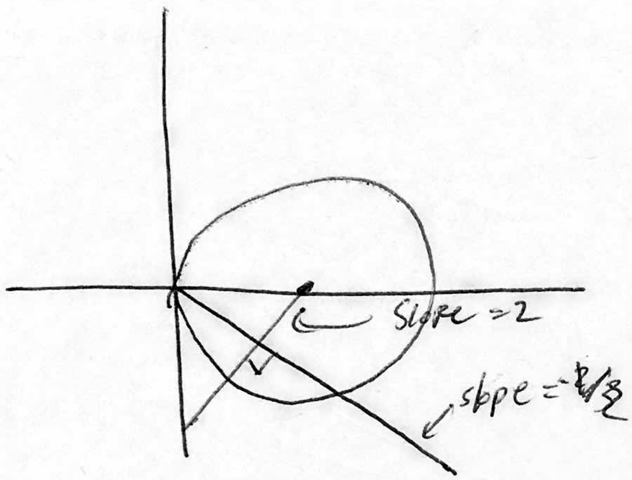
$\sqrt{4500} = r \cdot 90$

$r = \frac{\sqrt{4500}}{90}$

$d = r \cdot t \Rightarrow t = \frac{d}{r} = \frac{\sqrt{2000}}{\sqrt{4500}/90}$

$= 90\sqrt{\frac{4}{9}} = 60$

(c) (5 points) Where does the student come closest to the lamppost?



Slope = 2 pt = (25, 0)

$y = 2(x-25) = 2x - 50$

Intersect $\cancel{2(x-25)} =$

$2x - 50 = \frac{-1}{2}x$

$4x - 100 = -x$

$5x = 100$

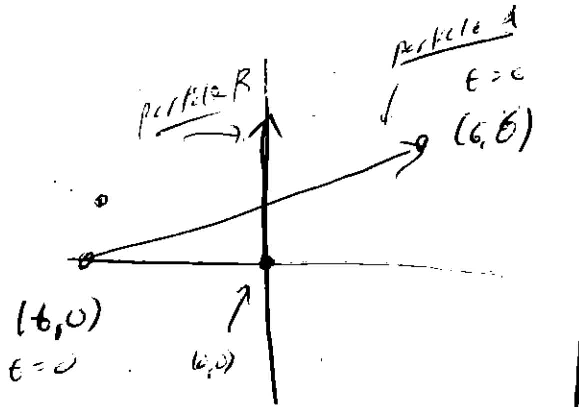
$x = 20$

$y = -10$

$(20, -10)$

3. The following problem has a fixed coordinate plane ruled in millimeters. Particle A travels in a straight line from coordinates $(-6, 0)$ towards the point $(6, 6)$, reaching it in 6 seconds. At the same time, a second particle (call it particle B), leaves from the origin and moves straight up along the y-axis traveling at 5 millimeters per second.

   (a) (5 points) Write parametric equations for each particle.



Particle A

$$x_A(t) = \frac{6-(-6)}{6-0}(t-0) - 6$$

$$= 2t - 6$$

$$y_A(t) = \frac{6-0}{6-0}(t-0) + 0 = t$$

Particle B

$$x_B(t) = 0$$

$$y_B(t) = 5t$$

   (b) (5 points) Write a function $d(t)$, where t is time, in seconds, and $d(t)$ is distance between the two particles, in millimeters.

$$d(t) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \sqrt{(2t-6)^2 + (t-5t)^2}$$

$$= \sqrt{4t^2 - 24t + 36 + 16t^2}$$

$$= \sqrt{20t^2 - 24t + 36}$$

(c) (5 points) What is the minimum distance between the two particles?

$d(t) = \sqrt{20t^2 - 24t + 36}$

min @ $\frac{-b}{2a} = \frac{24}{40} = \frac{3}{5}$ seconds

$d(\frac{3}{5}) = \sqrt{\cancel{20}^{4} \cdot \frac{9}{\cancel{25}_{5}} - 24(\frac{3}{5}) + 36}$

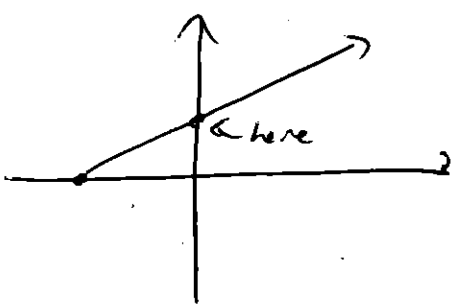
$= \sqrt{\frac{36}{5} - \frac{72}{5} + \frac{180}{5}}$

$= \sqrt{\frac{144}{5}} \approx 5.37$ mm

Dont plug into D?
-2

Dont take $\sqrt{\ }$
-1

(d) (5 points) The two paths cross at some point. Which point is this?



when $x_A(t) = 0$

$2t - 6 = 0$

$t = 3$

$y_A(3) = 3$

$x_A(3) = 0$

So $(3, 0)$

4. Let $f(x)$ be given by the following multipart rule.

$$f(x) = \begin{cases} 2x+5 & x \leq 0 \\ x^2 - 2x + 5 & x \geq 0 \end{cases}$$

(a) (5 points) Sketch a graph of $y = f(x)$. (HINT: You'll need to put the second part in vertex form to graph it).

$x^2 - 2x + 5$

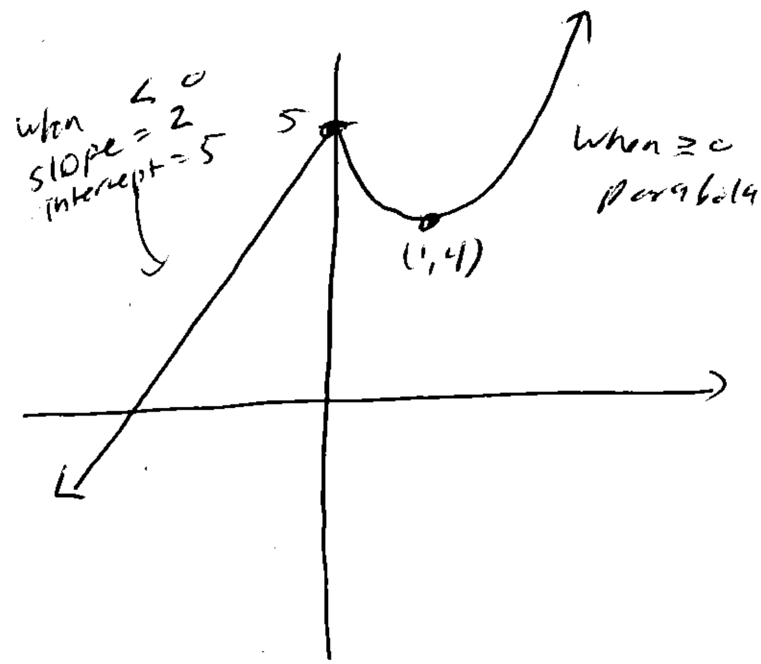
vertex

$h = -b/2a = \frac{2}{2} = 1$

$f(1) = 1 - 2 + 5$

$= 4$

$(1, 4$



(b) (5 points) Find all x such that $f(x) = 8$.

$8 = 2x + 5 \Rightarrow x = \frac{+3}{2} > 0$ so not in domain

(or use graph).

I would say...

$8 = x^2 - 2x + 5 \Longrightarrow x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$

or

$x = -1 \leftarrow$ not in domain

Only this

$\boxed{\text{Checking in D is 3 pts}}$