Math 120 B Quiz 4 (20 minutes) May 4, 2016

Name: Key

Answer the questions in the spaces provided. Show all necessary work. If you run out of space, use the back side and leave a message to indicate that you have done so. If you have any questions, raise your hand and I will come try to answer.

1. The city of Gotham has a serious pest problem.

   (a) (5 points) The number of rats in the city doubles every 2 years. Last year there were ten million rats. Write a function $r(t)$ which gives the number of rats t years from now. (Note, last year is $t = -1$).

$r(t) = A_0 b^t$

2 pts

(1) $(-1, 10,000,000) \Rightarrow 10,000,000 = A_0 b^{-1}$ (×2)

(2) $(1, 20,000,000) \Rightarrow 20,000,000 = A_0 b^1$

(2) ÷ (1) (×2)

$b^2 = 2$

$b = \sqrt{2}$

Plug into (1)

$10,000,000 = A_0 \cdot \frac{1}{\sqrt{2}}$

$A_0 = 10,000,000\sqrt{2}$

So $r(t) = 10,000,000\sqrt{2}\left(\sqrt{2}\right)^t$ (+1)

   (b) (5 points) After a failed experiment by the Gotham military scientists, 100,000 super cockroaches were released into the city last year ($t = -1$). This year the population has exploded to and there are 500,000 cockroaches in the city. Write a function $c(t)$ for the number of cockroaches t years from now.

2 pts

(1) $(-1, 100\,000) \Rightarrow 100\,000 = A_0 b^{-1}$

(2) $(0, 500\,000) \Rightarrow A_0 = 500\,000$ (×2)

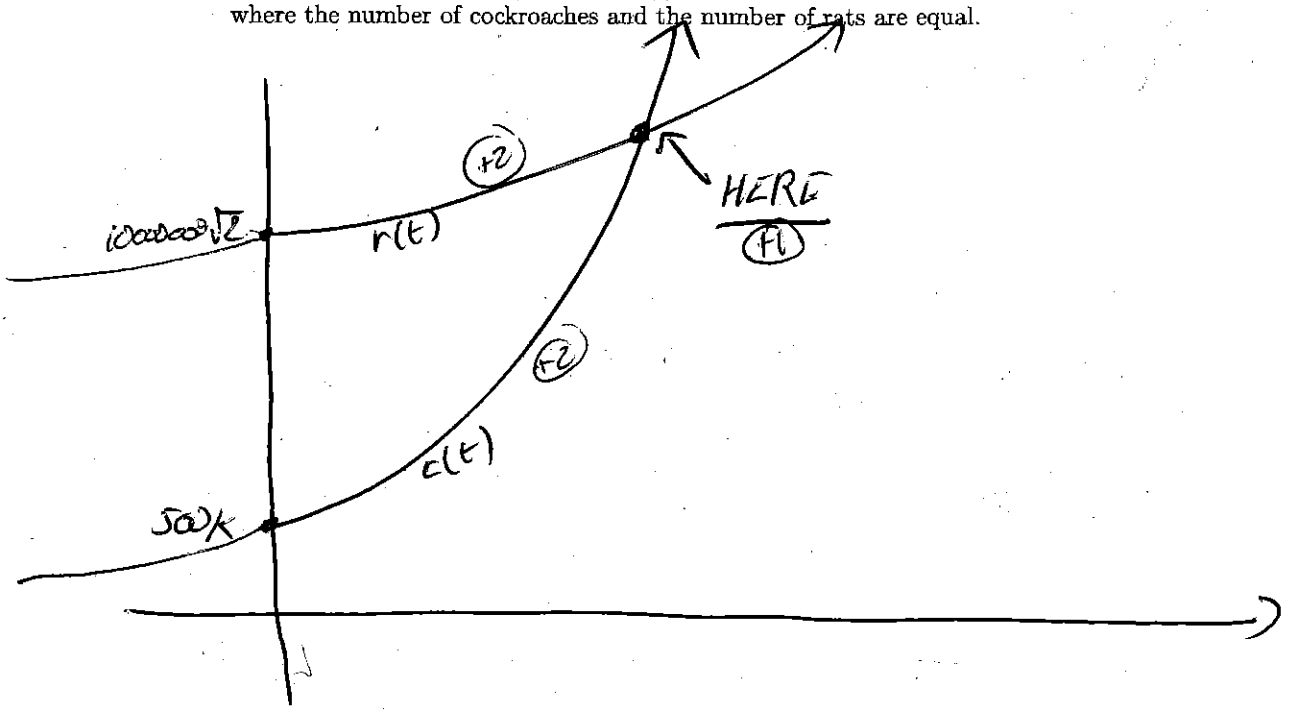
Plug A_0 into (1) (×2)

$100\,000 = \frac{500\,000}{b}$

$b = 5$

So $c(t) = 500\,000(5)^t$ (+1)

(c) (5 points) Sketch $y = r(t)$ and $y = c(t)$ **in the same coordinate plane.** Indicate on the graph where the number of cockroaches and the number of rats are equal.



(d) (5 points) When will there be the same number of cockroaches and rats?

$10000000\sqrt{2}(\sqrt{2})^t = 500000(5)^t$ (+1)

$20\sqrt{2}(\sqrt{2})^t = 5^t$

Apply ln to both sides (+2)

$\ln(20\sqrt{2}(\sqrt{2})^t) = \ln(5^t)$

$\ln(20\sqrt{2}) + t\ln(\sqrt{2}) = t\ln 5$ (+1)

So $t = \dfrac{\ln(20\sqrt{2})}{\ln(\sqrt{5}) - \ln(\sqrt{2})} \approx 2.647$ yrs

+1 more point for algebra