Name: Key

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. Leave your answers in *exact form* or round to 4 decimal places.

1. Pictured below is a new prototype for a compound unicycle, built for speed. The gear attached to the pedals is located at the top, and tire is the large wheel at the very bottom. Each of the central gears has a radius of 2 inches. The larger gears have radii of 4, 6, 8, and 10 inches as you go down.

   (a) (10 points) If you pedal at 1 rotation per second, how fast does the unicycle travel?

   (b) (10 points) A pebble is stuck against the bottom of the tire. Mark on the picture where on the tire the pebble will be after 30 seconds. Explain your reasoning.
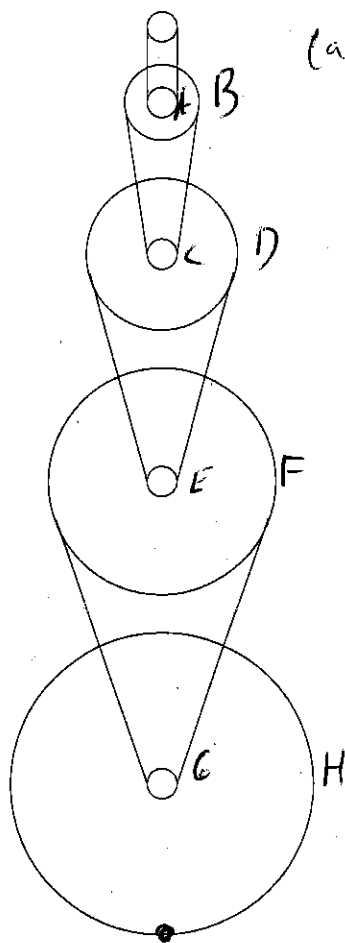
(a) $\omega_A = \omega_B = 1 \text{ RPS} = 2\pi \frac{rad}{sec}$

$v_C = v_B = 4 \cdot \omega_B = 8\pi \frac{rad}{sec}$

$\omega_D = \omega_C = \frac{8\pi}{2} = 4\pi$

$v_E = v_D = 6 \cdot 4\pi = 24\pi$

$\omega_F = \omega_E = \frac{24\pi}{2} = 12\pi$

$v_G = v_F = 8 \cdot 12\pi = 96\pi$

$\omega_H = \omega_G = \frac{96\pi}{2} = 48\pi \frac{rad}{sec}$

$v_H = 48\pi \cdot 10 = 480\pi \frac{in}{sec}$

$480\pi \frac{in}{sec} \left(\frac{1}{5280} \frac{mile}{ft} \right) \left(\frac{1 ft}{12 in} \right) \left(\frac{3600 sec}{1 hr} \right)$

$\approx 85.68 \text{ mph}$

(b) $\omega_H = 48\pi \frac{rad}{sec} = 24 \text{ rot/sec}$

After 30 seconds we've done $30 \cdot 24 = 720$ rotations so the pebble is at the bottom.

Name: *Key*

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1. A ship is sailing towards land, and sees a lighthouse in the distance, with it's light appearing at an angle of 2.5° above the ground. After travelling 1 mile, the light appears to form an angle of 4° above the ground.

   (a) How far is the ship from the lighthouse?

   (b) How tall is the lighthouse?

   (Hint: using trigonometric ratios you can set up two equations involving h and d and then solve the system).
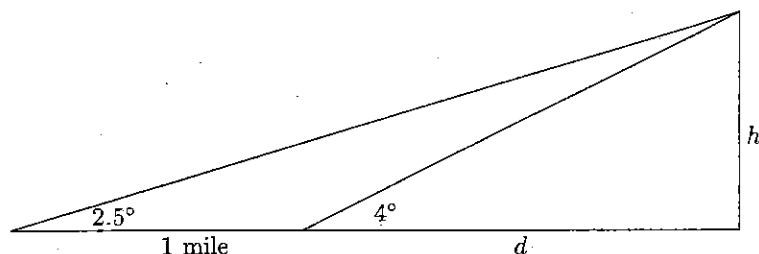
① $\tan(2.5°) = \dfrac{h}{d+1}$

$\tan(4°) = \dfrac{h}{d}$ 　(+6)

② Solve

$(d+1)\tan(2.5°) = d\tan(4°)$

$d(\tan(4°) - \tan(2.5°)) = \tan(2.5°)$

$d = \dfrac{\tan(2.5°)}{\tan(4°) - \tan(2.5°)}$ 　(+8)

≈ 1.6623 miles

③ Find h

$h = d\tan(4°)$

$= 1.6623(\tan(4°))$

$\approx .1162$ miles

≈ 613.73 ft

(+6)