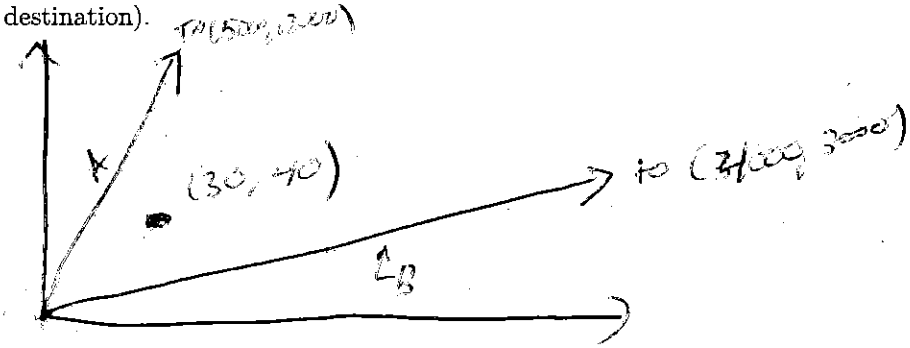
Name: Key

> Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. Leave your answers in *exact form* or round to 4 decimal places.

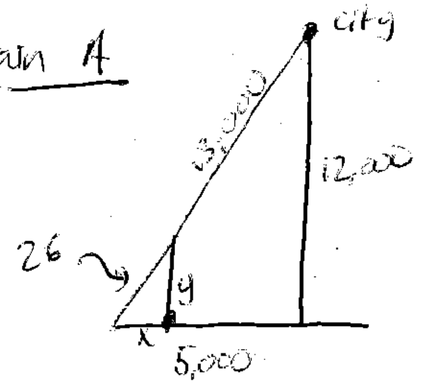
1. Two trains leave the same station at the same time. The first (train A) heads towards a city 5 kilometers east and 12 kilometers north of the station, at 26 meters per second. The second (train B) heads towards a town 4 kilometers east and 3 kilometers north at 20 meters per second. 30 meters east and 40 meters north of the station stands an engineer, watching the trains.

   (a) (3 points) Choose a coordinate system and draw a picture, labeling the tracks of each train and the engineer. (WARNING: it will be hard to put the destinations in the picture while keeping everything to scale. Instead, just draw the railroad tracks, understanding that the go until they reach the destination).



   (b) (5 points) What are the coordinates of each train after 1 second?

Train A



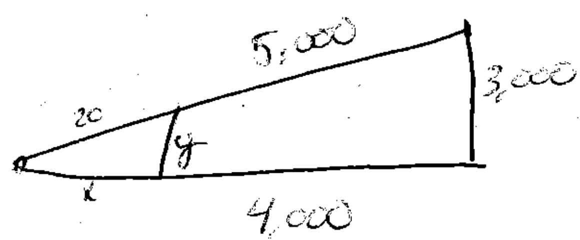
After 1 second, train A has travelled 26 meters.

So $\dfrac{x}{26} = \dfrac{5{,}000}{13{,}000}$ $\quad$ $\dfrac{y}{26} = \dfrac{12{,}000}{13{,}000}$

$x = 10$ $\quad$ $y = 24$

$\boxed{A \quad (10, 24)}$

Train B



After 1 second train B has gone 20 meters

So $\dfrac{x}{20} = \dfrac{4{,}000}{5{,}000}$ $\quad$ $\dfrac{y}{20} = \dfrac{3000}{5000}$

$x = 16$ $\quad$ $y = 12$

$\boxed{B \quad (16, 12)}$

(c) (6 points) Use part (b) to find parametric equations of motion for each train, with respect to a time variable t in seconds.

$$x_A(t) = \frac{\Delta x}{\Delta t}t + x_A(0) = \frac{10}{1}t + 0 = 10t$$

$$y_A(t) = \frac{\Delta y}{\Delta t}t + y_A(0) = \frac{24}{1}t + 0 = 24t$$

$$x_B(t) = \frac{\Delta x}{\Delta t}t + x_B(0) = \frac{16}{1}t + 0 = 16t$$

$$y_B(t) = \frac{\Delta y}{\Delta t}t + y_B(0) = \frac{12}{1}t + 0 = 12t$$

Train A:
$x_A(t) =$ $10t$
$y_A(t) =$ $24t$
Train B:
$x_B(t) =$ $16t$
$y_B(t) =$ $12t$

(d) (3 points) Write down a function $d_A(t)$ for the distance of train A to the engineer after t seconds. Do the same with $d_B(t)$ for train B.

$$d_A(t) = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(10t-30)^2 + (24t-40)^2}$$

$$d_B(t) = \sqrt{(16t-30)^2 + (12t-40)^2}$$

(e) (3 points) Let a new function be defined by the following rule: $f(t) = d_A(t) - d_B(t)$. In words, interpret this function. What does it mean when $f(t) > 0$? What about when $f(t) < 0$? When $f(t) = 0$?

$f(t) =$ (dist A from engineer) $-$ (dist B from engineer)

= How much further A is to engineer than B is.

$f(t) > 0 \implies$ B is closer to engineer

$f(t) < 0 \implies$ A is closer to engineer.

$f(t) = 0 \implies$ They are the same distance from the engineer.