Math 120 B Quiz 1 (20 minutes) April 6, 2016

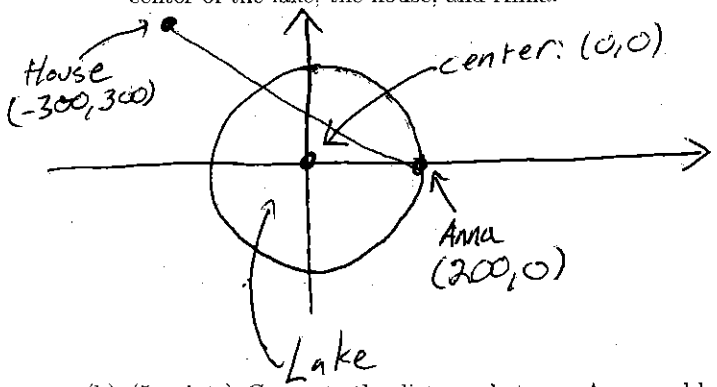
Name: Key

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. ~~Leave your answers in exact form instead of decimal approximations.~~

Round to 3 decimal places.

1. Anna's house located exactly 300 meters west and 300 meters north from the center of a of a circular lake, whose radius is 200 meters. Anna is on the eastern shore of the lake.

   (a) (5 points) Draw a picture of the lake and impose a coordinate system. Label the coordinates of the center of the lake, the house, and Anna.



   (b) (5 points) Compute the distance between Anna and her house.

$$D = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(-300-200)^2 + (300-0)^2}$$

$$= \sqrt{(-500)^2 + (300)^2} \approx 583.095$$

   (c) (5 points) Anna got hungry, and decided to head directly home. She paddled her canoe in the direction of her house. Find the equation for the line along which she paddles. (Hint: start by finding the slope, and use the coordinates from part (a) to use point slope form).

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{300}{-500} = \frac{-3}{5}$$

$$y = \frac{-3}{5}(x - 200) + 0$$

$$= \frac{-3}{5}x + 120$$

(d) (5 points) After canoeing across along this line, what are the coordinates of the point where she reached the shore?

Intersect circle & line

① $x^2 + y^2 = 200^2$

② $y = \frac{-3}{5}x + 120$

Plug ② into ①

$x^2 + \left(\frac{-3}{5}x + 120\right)^2 = 200^2$

$x^2 + \frac{9}{25}x^2 - 144x + 14400 - 40000 = 0$

simplify $\frac{34}{25}x^2 - 144x - 25600 = 0$ (*)

(*) continue

Quadratic formula

$x = \frac{144 \pm \sqrt{(-144)^2 - 4\left(\frac{34}{25}\right)(-256}}{2\left(\frac{34}{25}\right)}$

$x = 200$ or $x \approx -94.118$

already got.

$y = \frac{-3}{5}(-94.118) + 12$

$= 176.471$

$P = (-94.118, 176.47$

(e) **(BONUS: 5 points)** Suppose she canoes at speed of 2 meters per second and walks at a speed 3 meters per second.

① Distance in water (-94.118, 176.471) & (200, 0)

$D = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(200 + 94.118)^2 + (0 - 176.471)^2}$

$= 342.998$

② Distance on land $= 583.095 - 342.998$

$= 240.097$

Time = (Time on water) + (time on land)

$= \frac{342.998\text{ m}}{2\text{ m/s}} + \frac{240.097\text{m}}{3\text{ m/s}} = 251.531$ seconds.