Math 120 B | Midterm 1 (80 minutes) | April 21, 2016

Name: Key

Directions:

- You have 80 minutes to complete this exam.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet (so two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you have any questions, raise your hand.

Question	Points	Score
1	15	
2	20	
3	15	
4	10	
5	15	
Total:	75	

1. (15 points) An ant is located at (6,-2) and begins walking towards a sugar cube at (0,4) arriving there in 2 seconds. Meanwhile, a spider at the origin begins its hike towards its web at (4,3) at a rate of 5 inches per second. (NOTE: coordinates are in inches)

(a) Write parametric equations of motion for the spider and the ant.

Ant

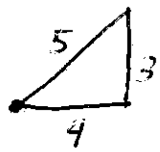
$x_{ant}(t) = \frac{\Delta x}{\Delta t} t + x_0$

$= -3t + 6$

$y_{ant}(t) = \frac{\Delta y}{\Delta t} t + y_0$

$= 3t - 2$

Spider



$d = r \cdot t$

so $\Delta t = \frac{d}{r} = \frac{5}{5} = 1$

so $x_s(t) = 4t$

$y_s(t) = 3t$

2 for each thing...

(b) Write down a function $d(t)$ which measures the distance between the spider and the ant after t seconds. When are they closest together? How close are they at this point?

$d(t) = \sqrt{(x_a(t) - x_s(t))^2 + (y_a(t) + y_s(t))^2}$

$= \sqrt{(-7t+6)^2 + (-2)^2} = \sqrt{49t^2 + 84t + 40}$ +2

Max at $t = \frac{-b}{2a} = \frac{84}{98} \approx 0.857$ +2

$d\left(\frac{84}{98}\right) = 2$ +1

(c) The path of the ant and the path of the spider cross at some point. Where is this point?

m_a = Slope for ant: $\frac{\Delta y}{\Delta x} = \frac{-6}{6} = -1$

intercept for ant, $b = 4$)

Ant: $y = -x + 4$

Slope for spider: $m_s = \frac{3}{4}$

intercep: $b = 4$

+3

Spider: $y = \frac{3}{4}x$

Intersect

$\frac{3}{4}x = -x + 4$

$3x = -4x + 16$

$7x = 16$ +2

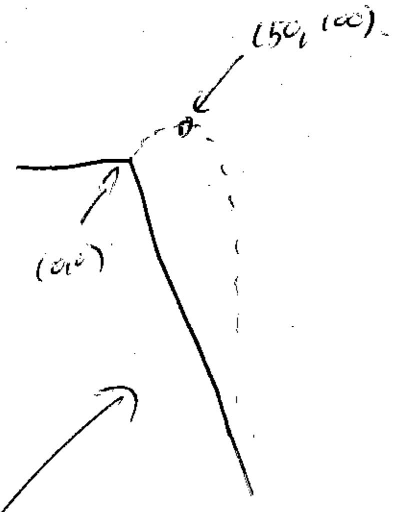
$x = \frac{16}{7}$

$y = \left(\frac{3}{4}\right)\left(\frac{16}{7}\right) = \frac{12}{7}$

Point is $\left(\frac{16}{7}, \frac{12}{7}\right)$.

2. (20 points) I am standing at the top of a hill of constant slope, and I throw a rock. 50 feet out, the rock reaches its maximum height of 100 feet above where I threw it from.

   (a) Suppose that where I stand is at the origin, and the positive x-direction is the direction the rock is thrown. Write a function $y = f(x)$ representing the path of the rock.



We know:

$y = a(x-50)^2 + 100$ (+2) (vertex form)

Also, $f(0) = 0$

$0 = a(-50)^2 + 100$

$2500a = -100$

$a = \frac{-1}{25}$ (+3)

so $f(x) = \frac{-1}{25}(x-50)^2 + 100$

   (b) The hill is quite steep, descending 1 foot for each horizontal foot. Write an equation for the location of the ground (with $x > 0$). With this, write a new function $y = h(x)$ representing the height of the rock above the ground.

~~gr~~ Slope $= -1$

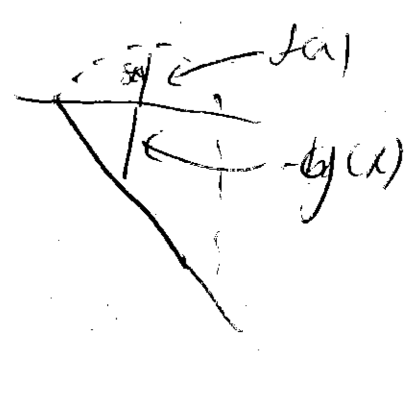
Intercept $= 0$ (+2)

ground $y = g(x) = -x$

above thrown point

height $= f(x) - g(x)$ ← extra ground distance

$\boxed{h(x) = \frac{-1}{25}(x-50)^2 + 100 + x}$ (+3)



(c) What is the maximum height of the rock off the ground? At which point (x, y) is this height reached?

$$\text{height} = h(x) = \frac{-1}{25}(x-50)^2 + 100 + x$$
$$= \frac{-1}{25}(x^2 - 100x + 2500) + 100 + x$$
$$= \frac{-1}{25}x^2 + 4x - 100 + 100 + x$$
$$= \frac{-1}{25}x^2 + 5(x)$$

Max @ $-b/2a = \frac{-5}{-2/25} = \frac{125}{2} = 62.5$ +3

$\boxed{\text{height} = h\left(\frac{-b}{2a}\right) = \frac{625}{4} = 156.25}$ +1

$y\text{-coord} = f\left(\frac{-b}{2a}\right) = \frac{375}{4} = 93.75$ +1.

(d) At which point (x, y) does the rock land?

So $h(x) = 0$ +1 $\quad h(x) = \frac{-1}{25}x(x-125)$

$$\frac{-1}{25}x(x-125) = 0$$

$x = 0$

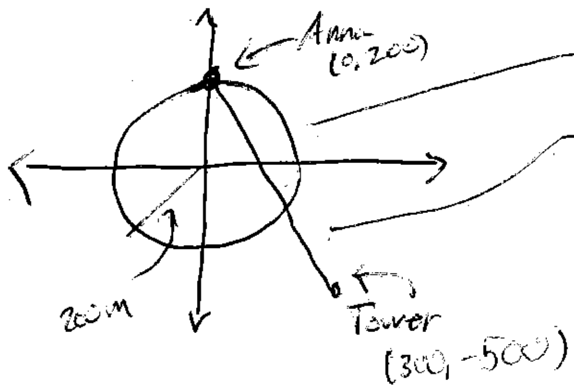
or $\boxed{x = 125}$ +3 ← this one.

$y\text{-coord} = f(125) = -125$ +1

$(125, -125)$.

3. (15 points) Anna (from the first quiz) is out canoeing again. Today she relaxes on the north bank of the same lake (of radius 200m). She begins canoeing in the direction of a point 500 meters south and 300 meters east of the center of the lake.

(a) Choose a coordinate system and draw a picture. Label Anna, the center of the lake, and the tower, and the path she travels. Write down equations for the edge of the lake and for Anna's path.



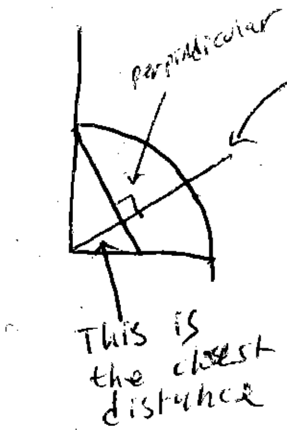
Lake: $x^2 + y^2 = 200^2$

slope: $\frac{\Delta y}{\Delta x} = \frac{200+500}{0-300} = \frac{-7}{3}$

intercept: 200

$y = \frac{-7}{3}x + 200$

(b) How close does Anna come to the center of the lake?



Slope = negative inverse = $3/7$

Intercept = 0

$y = \frac{3}{7}x$

Intersect w/ annas path

$\frac{3}{7}x = -\frac{7}{3}x + 200$

$\frac{58}{21}x = 200$

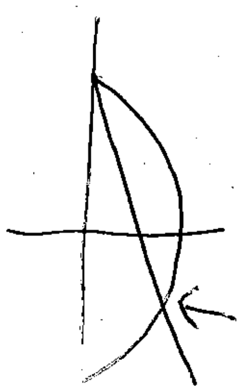
$x = 72.41$

$y = \frac{3}{7}(x) = 31.03$

$d = \sqrt{x^2 + y^2}$

≈ 78.79

(c) Where does Anna exit the lake?



$x^2 + (\frac{-3}{7}x + 200)^2 = 200^2$ +2

$x^2 + \frac{9}{49}x^2 - \frac{1200}{7}x + \cancel{200^2} = \cancel{200^2}$

$x\left(\frac{58}{49}x - \frac{1200}{7}\right) = 0$

$\frac{58}{49}x = \frac{1200}{7}$ +2

$x = 144.83$

$y = \frac{-7}{3}x + 200$ +1

$= -137.93$

$(144.83, -137.93)$

4. (10 points) Let f be a function. Consider the expression:

$$\frac{f(x+h)-f(x)}{h}.$$

Notice that if we plug in $h=0$ we get:

$$\frac{f(x+0)-f(x)}{0}=\frac{0}{0},$$

which doesn't make sense. Sometimes, though, we can plug in $x+h$ and x and the h on the bottom will cancel out, allowing us to plug in $h=0$ after some work. For example, if $f(x)=x$ then:

$$\frac{f(x+h)-f(x)}{f(x)}=\frac{x+h-x}{h}=\frac{h}{h}=1.$$

Do this for the following two functions:

(a) $f(x)=14x+7$.

$$\frac{f(x+h)-f(x)}{h}=\frac{14(x+h)+7-14x-7}{h}=\frac{14x+14h+7-14x-7}{h}$$

$$=\frac{14\cancel{h}}{\cancel{h}}=14$$

+2 +1 +1 +2

(b) $g(x)=7x^2+7x+9$

$$\frac{g(x+h)-g(x)}{h}=\frac{7(x+h)^2+7(x+h)+9-7x^2-7x-9}{h}$$

$$=\frac{\cancel{7x^2}+14xh+7h^2+\cancel{7x}+7h+\cancel{9}-\cancel{7x^2}-\cancel{7x}-\cancel{9}}{h}$$

$$=\frac{\cancel{h}(14x+7h+7)}{\cancel{h}}=14x+7h+7$$

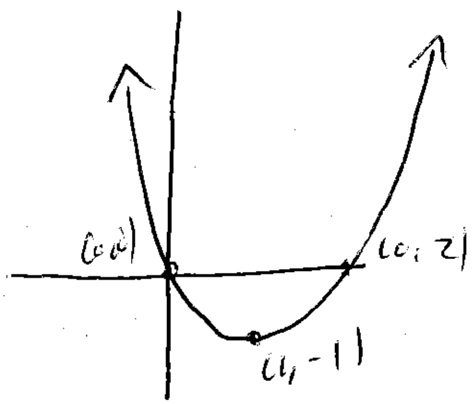
$$\overset{h\to 0}{=}14x+7$$

+2 +1 +2

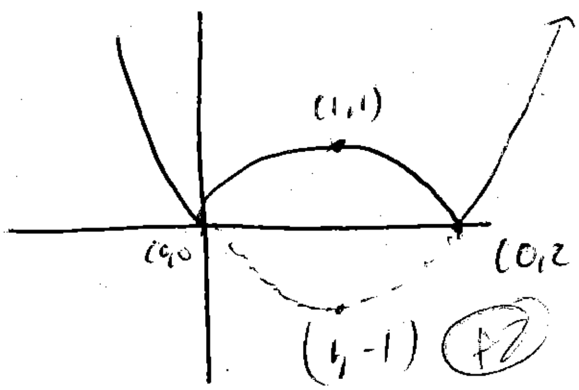
roots $\qquad$ vertex form

5. (15 points) Let $f(x) = x^2 - 2x$. $= (x-2)x = (x-1)^2 - 1$

(a) Draw the graph of $y = f(x)$.



(b) Draw $y = |f(x)|$ and write its multi part rule.



$$y = \begin{cases} f(x) & x \le 0 \text{ or } x \ge 2 \\ -f(x) & 0 \le x \le 2 \end{cases}$$

+3

$$= \begin{cases} x^2 - 2x & x \le 0 \text{ or } x \ge 2 \\ -x^2 + 2x & 0 \le x \le 2 \end{cases}$$

+1

(c) Let $g(t) = t - 5$. What is $f(g(t))$? Draw it.

$$f(g(t)) = (t-5)^2 - 2(t-5)$$
$$= t^2 - 10t + 25 - 2t + 10$$
$$= t^2 - 12t + 35$$

+2