Math 120 KAUST Midterm Exam (90 minutes) September 17th, 2018

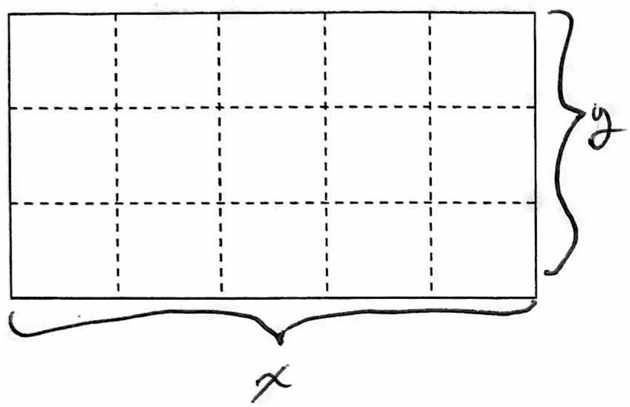
Name: Key

Directions:

- You have 90 minutes to complete this exam.
- There are 5 problems on 9 pages.
- No graphing calculators are allowed.
- You are allowed one hand-written sheet (two sided is ok) of notes on regular 8.5-11 paper.
- You must show ALL your work.
- Leave answers in EXACT FORM or record up to 2 DECIMAL PLACES.
- If you need extra room, use the back side of the page. Include a note to indicate to the reader that you have done so.
- If you have any questions, raise your hand.

Question	Points	Score
1	10	
2	20	
3	15	
4	20	
5	15	
Total:	80	

1. (10 points) You would like to build a rectangular enclosure partitioned into a grid, as pictured below. The exterior fencing (solid lines) costs \$25 a foot, and the fencing for the interior partitions (dashed lines) cost \$12 a foot. If you have \$11500 to spend, what is the maximum area you can enclose?



Area $= xy$

Constraint $2\cdot 25\cdot x + 2\cdot 12\cdot x + 2\cdot 25\cdot y + 4\cdot 12\cdot y = \text{cost}$

$$74x + 98y = 11500$$

$$y = \frac{11500 - 74x}{98}$$

$$\text{Area} = x\cdot\left(\frac{11500-74x}{98}\right) = \frac{11500}{98}x - \frac{74}{98}x^2$$

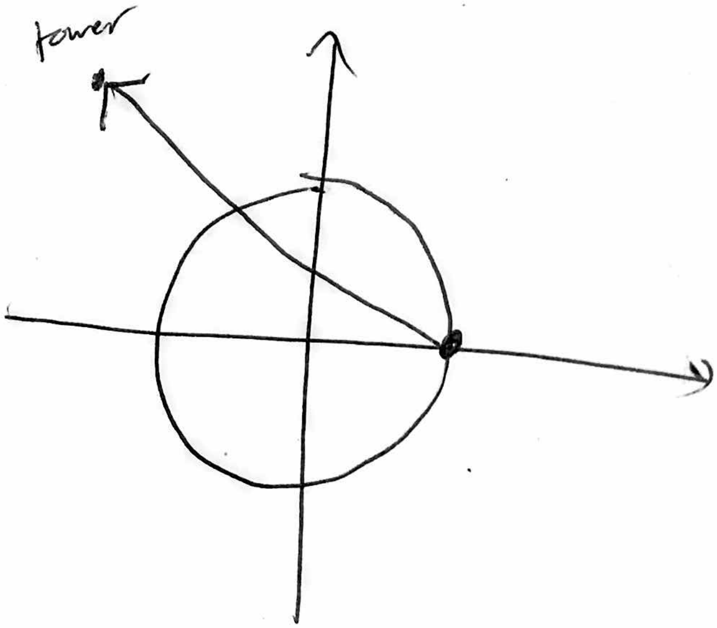
Max @ $\frac{-b}{2a} = \frac{-11500/98}{-2\cdot 74/98} \approx 77.70$

$$y = \frac{11500 - 74(77.70)}{98} \approx 58.67$$

$$\text{Area} = xy = 4558.93$$

2. There is a perfectly circular lake with a 1 km radius. Jimi sits on the eastern shore when he spots a tower across the lake. He immediately begins swimming towards it a rate of 2 m/s. Suppose the tower is exactly 2 ~~miles~~ km west and 2 ~~miles~~ km north of the center of the lake.

(a) (5 points) Set coordinates. Draw the lake, the tower, and label Jimi's path.



(b) (5 points) Where does Jimi exit the lake?

Jimi's Path

Point = $(1,0)$

Slope = $\frac{-2}{3}$

$y = \frac{-2}{3}(x-1)$

$= \frac{-2}{3}x + \frac{2}{3}$

+2

Intersect w/ $x^2+y^2=1$ ← lake

$x^2 + \left(\frac{-2}{3}x + \frac{2}{3}\right)^2 = 1$

$x^2 + \frac{4}{9}x^2 - \frac{8}{9}x + \frac{4}{9} = 1$

$\frac{13}{9}x^2 - \frac{8}{9}x - \frac{5}{9} = 0$

clear

$13x^2 - 8x - 5 = 0$

$(x-1)(13x+5) = 0$

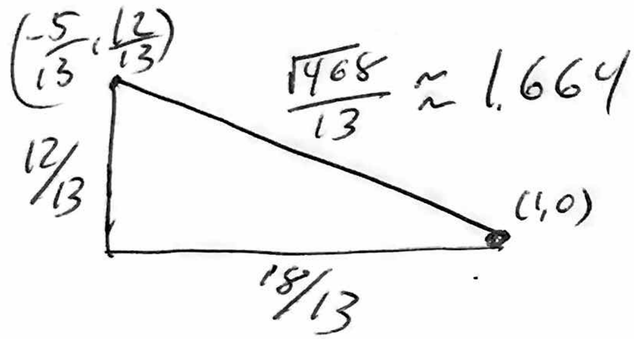
$x = \frac{-5}{13} \Rightarrow y = \frac{12}{13}$

$\left(\frac{-5}{13}, \frac{12}{13}\right)$

+3

(c) (5 points) How long is Jimi swimming? +3

Step 1: distance



Step 2: time +2

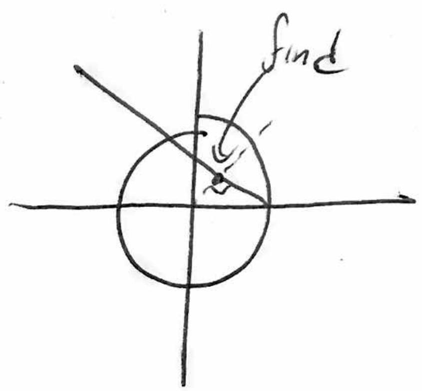
$d = r \cdot t$

$1.664\,km = 2\frac{m}{s} \cdot t$

$1664\,m = 2\frac{m}{s} \cdot t$

$t \approx 832$ sec

(d) (5 points) What is the closest distance Jimi comes to the center of the lake.



Perpendicular line

Slope: $\frac{1}{-2/3} = \frac{3}{2}$

Point: $(0,0)$

1) $y = \frac{3}{2}x$

2) $y = \frac{-2}{3}x + \frac{2}{3}$

$\frac{3}{2}x = \frac{-2}{3}x + \frac{2}{3}$

$\frac{9}{6}x = \frac{-4}{6} + \frac{4}{6}$

$\frac{13}{6}x = \frac{4}{6}$

$13x = 4$

$x = 4/13$

$y = \frac{3}{2}x = 6/13$

+3

$d = \sqrt{\left(\frac{4}{13}\right)^2 + \left(\frac{6}{13}\right)^2}$ +2

$\approx .5547$

$\approx 554.7\,m$

3. A time-destroying dictator has ordered the destruction of every watch, clock, calendar and sundial. Using your quick wit, you devise a clever device to keep track of time. Your invention involved setting aside 2000g of Temporium, a mildly radioactive compound with a half life of 112 days, in a controlled environment.

(a) (5 points) Write a function $W(t)$, which returns the weight of the Temporium after t days. Carefully state the domain and range.

$W(t) = 2000(\frac{1}{2})^{t/112}$

Domain $t \geq 0$

Range $2000 \geq W > 0$

(b) (5 points) Finish your radioactive clock by inverting the function from part (a). Explain in words what this function does (i.e., what is the input? What is the output?). As usual, carefully state the domain and range.

Solve for t

$\frac{w}{2000} = (\frac{1}{2})^{t/112}$

$\ln(w/2000) = \frac{t}{112} \ln(1/2)$

$t = 112 \frac{\ln(w/2000)}{\ln(1/2)}$

Input: Weight remaining

Output: How long it's been

D: $0 < w \leq 2000$

R $t \geq 0$.

(c) (5 points) Rumors of your secret clock land you in jail, but it was never discovered. You escape after what feels like ages, and go to your clock to discover that 50mg of Temporium remain. How long has it been? (NOTE: 1g = 1000mg)

50mg = .05g

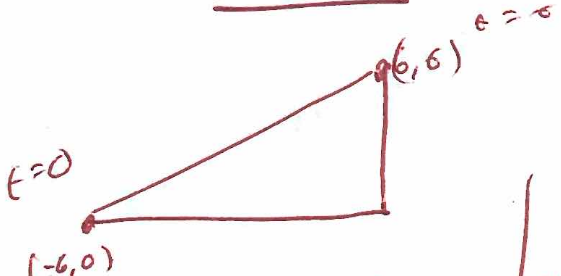
Find $t(.05) = 112 \frac{\ln(.05/2000)}{\ln(1/2)}$

$= \underline{1712.22 \text{ days}}$

4. The following problem has a fixed coordinate plane ruled in millimeters. Particle A travels in a straight line from coordinates $(-6, 0)$ towards the point $(6, 6)$, reaching it in 6 seconds. At the same time, a second particle (call it particle B), leaves from the origin and moves straight up along the y-axis traveling at 5 millimeters per second.

(a) (5 points) Write parametric equations for each particle.

Particle A



$x_A(t) = \frac{\Delta x}{\Delta t}(t - t_0) + x_0$

$= \frac{12}{6}(t) + 6$

$= 2t - 6$

$y_A(t) = \frac{\Delta y}{\Delta t}(t-6) + y_0$

$= \frac{6}{6}(t) + 0$

$= t$

$x_B(t) = 0$

$y_B(t) = 5t$

(b) (5 points) Write a function $d(t)$, where t is time, in seconds, and $d(t)$ is distance between the two particles, in millimeters.

$d(t) = \sqrt{\Delta x^2 + \Delta y^2}$

$= \sqrt{(2t-6)^2 + (t-5t)^2}$

$= \sqrt{4t^2 - 24t + 36 + 16t^2}$

$= \sqrt{20t^2 - 24t + 36}$

(c) (5 points) What is the minimum distance between the two particles?

$\underline{\text{min @}}$ $\frac{-b}{2a} = \frac{24}{40} = \frac{3}{5}$

$d(\frac{3}{5}) = \sqrt{\frac{144}{5}} \approx 5.37$ mm

(d) (5 points) The two paths cross at some point. Which point is this?

$\underline{x=0}$

$x_A(t) = 0$

$2t - 6 = 0$

$t = \cancel{3}\ 3$

$y_B(t) = 3$

$\rightarrow (0, 3)$

5. A parabola contains the three points, $(0, -12)$, $(2, 4)$, and $(5, -2)$.

(a) (5 points) Write the equation of the parabola in standard form $f(x) = ax^2 + bx + c$.

i) $-12 = a \cdot 0^2 + b \cdot 0 + c$

$\Rightarrow c = -12$

ii) $4 = 4a + 2b - 12 \Rightarrow 16 = 4a + 2b \Rightarrow b = 8 - 2a$

iii) $-2 = 25a + 5b - 12$

$\hookrightarrow 10 = 25a + 5(8 - 2a)$

$10 = 25a + 40 - 10a$

$-30 = 15a$

$a = -2$

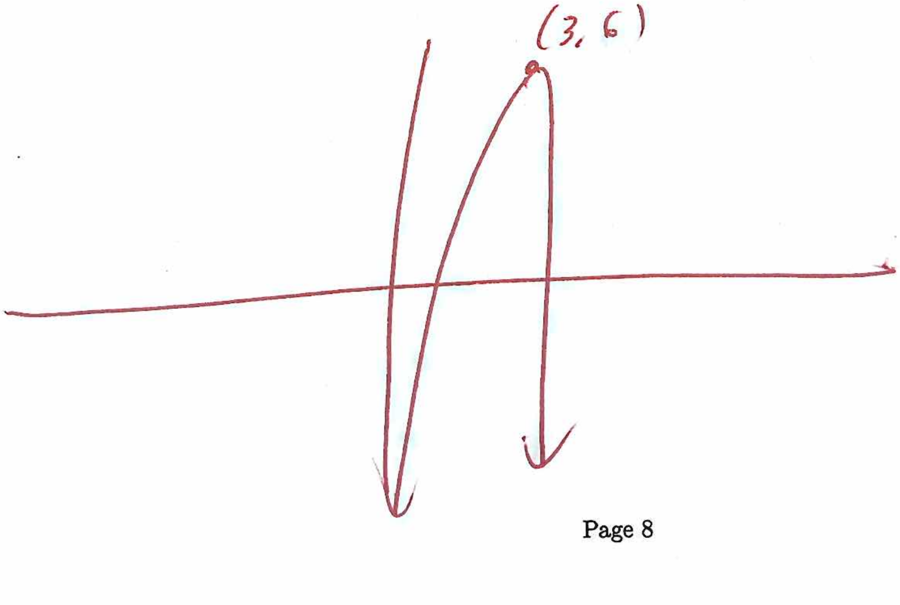
$b = 8 - 2a = 8 - 2(-2)$
$= 12$

$f(x) = -2x^2 + 12x - 12$

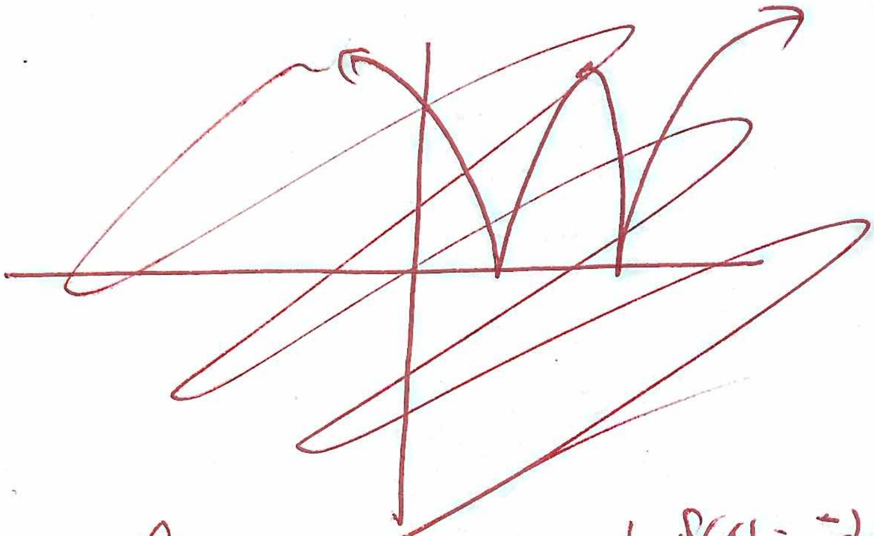
(b) (5 points) Convert $f(x)$ into vertex form $f(x) = a(x-h)^2 + k$ and then sketch a graph of $y = f(x)$, labeling the vertex.

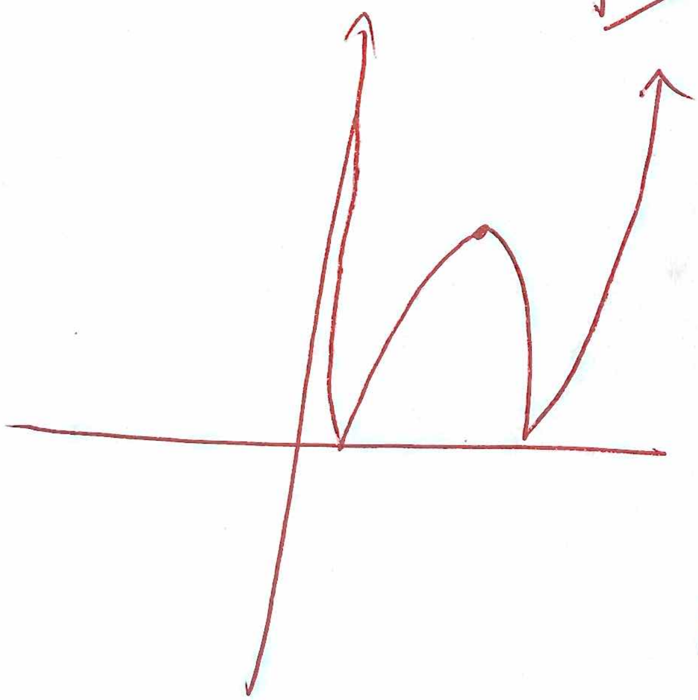
$h = -b/2a = \frac{-12}{-4} = 3$

$k = f(h) = f(3) = -2 \cdot 9 + 12 \cdot 3 - 12 = 6$



(c) (5 points) Sketch $y = |f(x)|$, and write the associated multipart rule.





$$f(x) = -2(x-3)^2 + 6 = 0$$

$$(x-3)^2 = 3$$

$$x = 3 \pm \sqrt{3}$$

$$|f(x)| = \begin{cases} -2x^2 + 12x - 12 & 3-\sqrt{3} \le x \le 3+\sqrt{3} \\ 2x - 12x + 12 & \text{else} \end{cases}$$