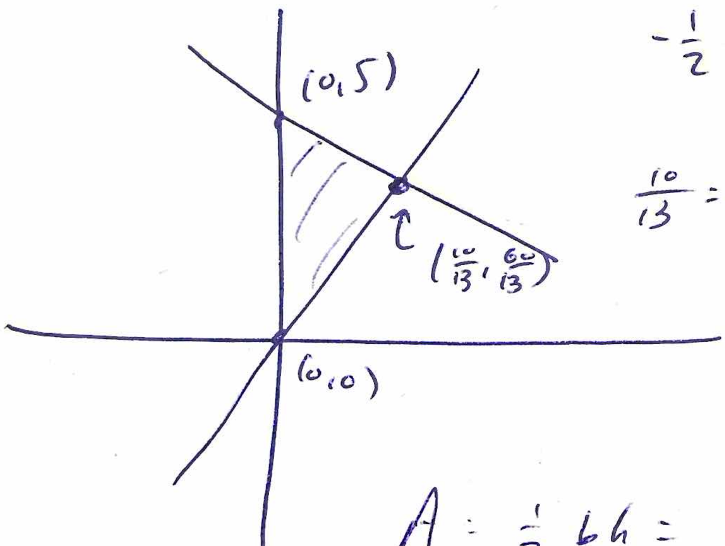
Math 120 KAUST | Worksheet 1 | September ~~5th~~ 6th, 2018

Name:

> Answer the questions in the spaces provided. Don't hesitate to ask me or your peers for help, this is not a quiz.

1. Working with lines.

   What is the area of the triangle determined by $y = -\frac{1}{2}x+5$, $y = 6x$ and the y-axis. (First graph the lines in a coordinate plane and shade the triangle you are studying. It may be useful to find the intersection points of the lines).



$-\frac{1}{2}x+5=6x$

$5=6.5x$

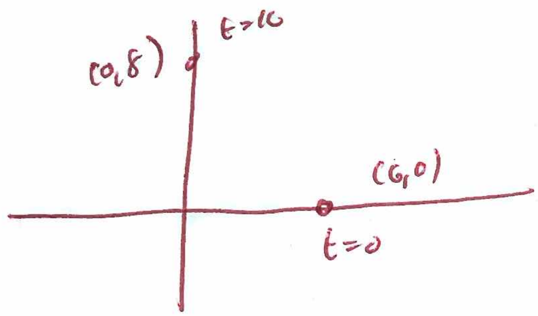
$\frac{10}{13}=\frac{5}{6.5}=x$

$A=\frac{1}{2}bh=\frac{1}{2}(5)\left(\frac{10}{13}\right)=\frac{25}{13}$

2. Parametrics

~~Juliet and Mercutio are moving at constant speeds in the xy-plane. They start moving at the same time. Juliet starts at the point $(0,-6)$ and heads in a straight line toward the point $(10,5)$, reaching it in 10 seconds. Mercutio starts at $(9,14)$ and moves in a straight line. Mercutio passes through the same point on the x-axis as Juliet, but 2 seconds after she does. How long does it take Mercutio to reach the y-axis? Write a function $d(t)$ that measures their distance apart after t seconds.~~

Spider: $(6,0) \longrightarrow (0,8)$ in 10 seconds



x: $(0,6)$ & $(10,0)$

$m = \frac{\Delta x}{\Delta t} = \frac{-6}{10} = \frac{-3}{5}$

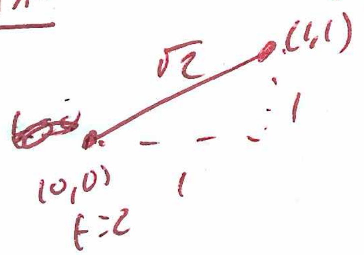
$x(t) = \frac{-3}{5}t + 6$

y: $(0,0)$ & $(10,8)$

$m = \frac{\Delta y}{\Delta t} = \frac{8}{10} = \frac{4}{5}$

$y(t) = \frac{4}{5}t$

Ant



x: $(2,0)$ & $(3,1)$

$m = \frac{\Delta x}{\Delta t} = 1$

$x(t) = (t-2) = t-2$

y: same $y(t) = t-2$

Do they collide?

x-coords match @

$t-2 = \frac{-3}{5}t + 6$

$\frac{8}{5}t = 8$

$t = 5$

y-coords match @

$t-2 = \frac{4}{5}t$

$\frac{1}{5}t = 2$

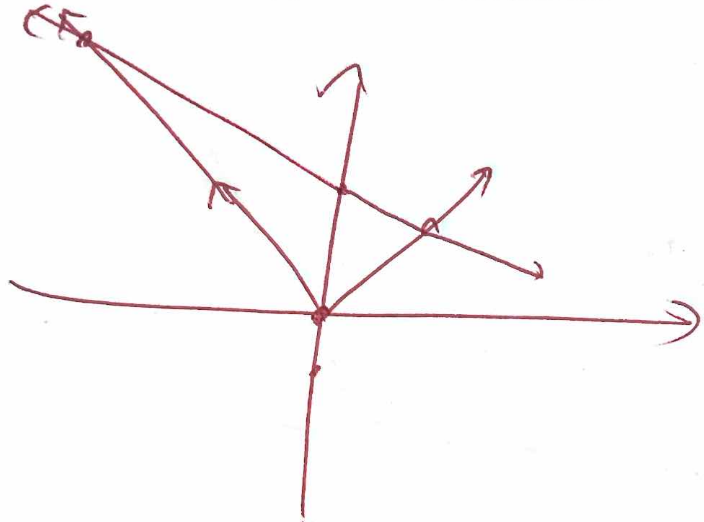
$t = 10$

not equal

So NO.

3. Functions Recall that $|x|$ represents the absolute value of x.

(a) Sketch a graph of the function $y = |x|$ and write its multipart rule.



$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

(b) In the same axis as above, sketch the like $y = -\frac{1}{2}x + 3$. Notice that the two curves intersect at two points. What are the coordinates of these two points?

$-\frac{1}{2}x + 3 = x$

$3 = \frac{3}{2}x$

$x = 2$

$(2, 2)$

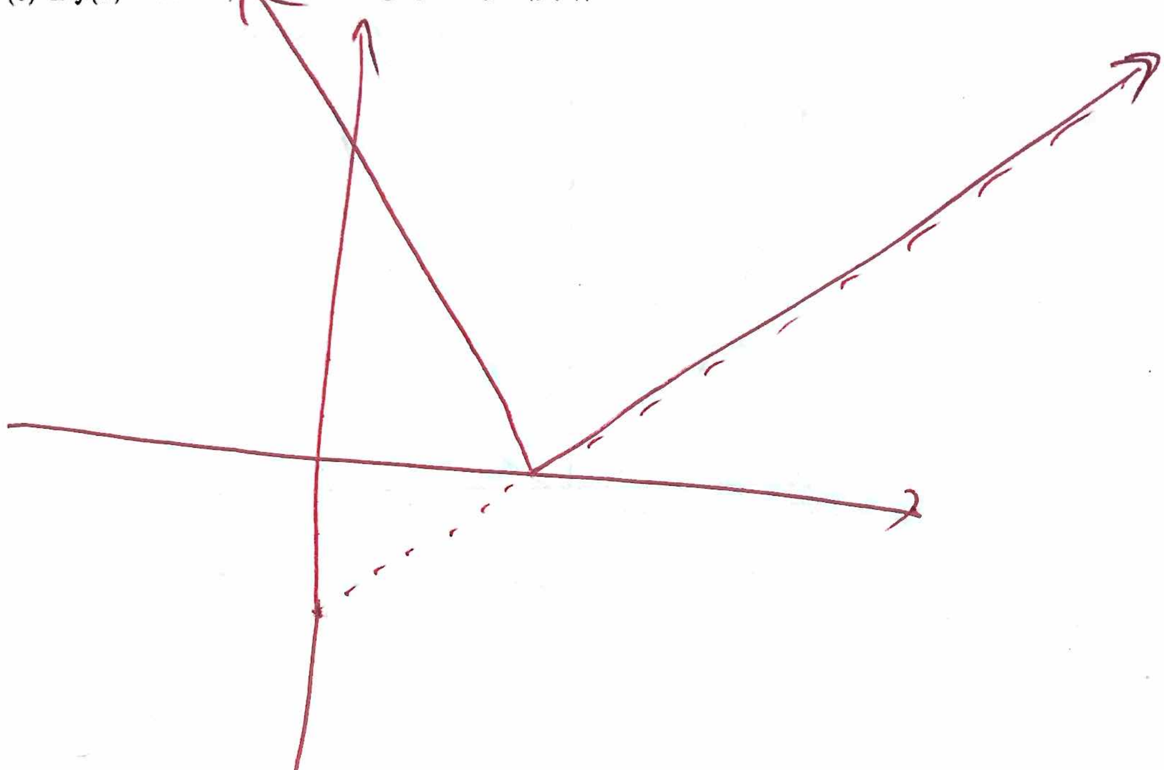
$-\frac{1}{2}x + 3 = -x$

$\frac{1}{2}x = -3$

$x = -6$

$(-6, 6)$

(c) If $f(x) = 2x - 2$, the sketch a graph of $y = |f(x)|$.



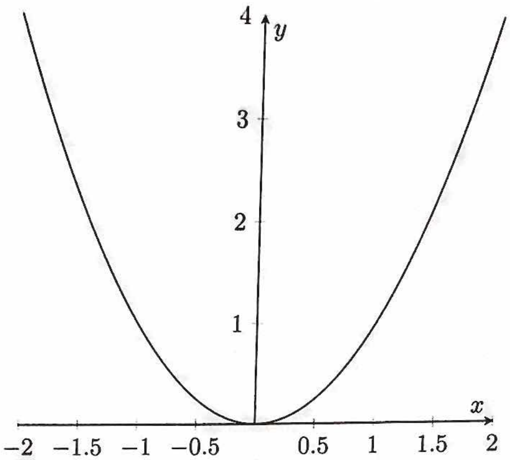
(d) Write a multi-part rule for $|f(x)|$.

$$|2x-2| = \begin{cases} 2x-2 & 2x-2 \geq 0 \\ -2x+2 & 2x-2 \leq 0 \end{cases}$$

$$= \begin{cases} 2x-2 & x \geq 1 \\ 2-2x & x \leq 1. \end{cases}$$

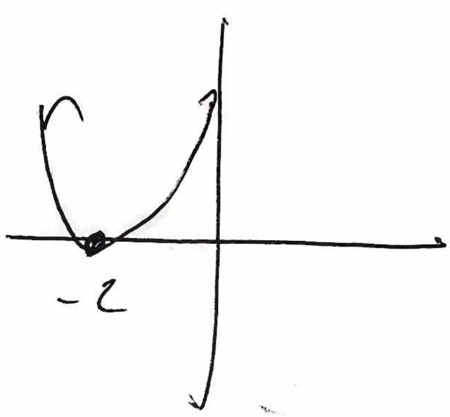
4. Moving stuff around.

The graph of the function $y = x^2$ looks as follows.

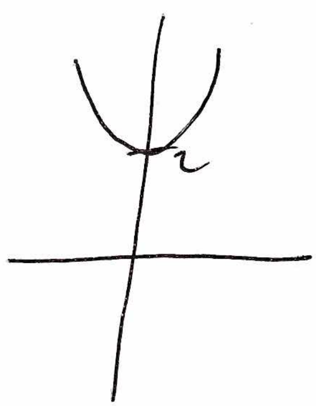


Sketch the graphs of $y = f(x+2)$, $y = f(x)+2$, $y = f(2x)$, $y = 2f(x)$.

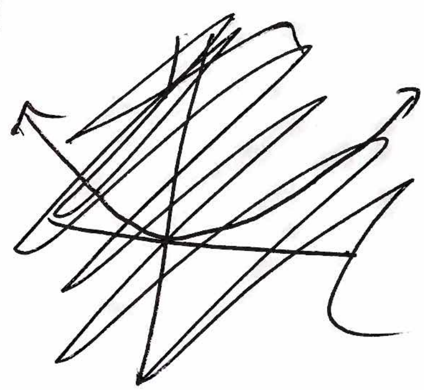
$f(x+2)$

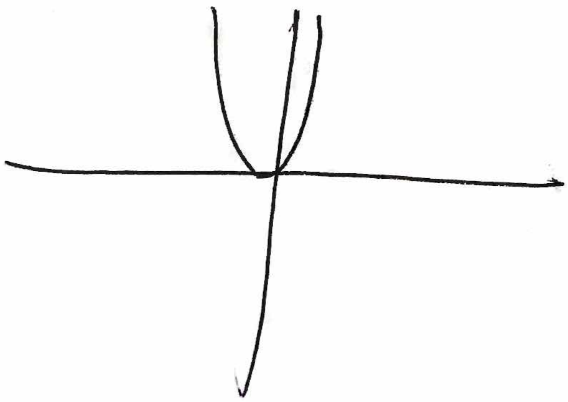


$f(x)+2$



$f(2x)$





Day 7

Example

Sketch $g(x) = \begin{cases} 1 & x \le -1 \\ 1+\sqrt{1-x^2} & -1 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$

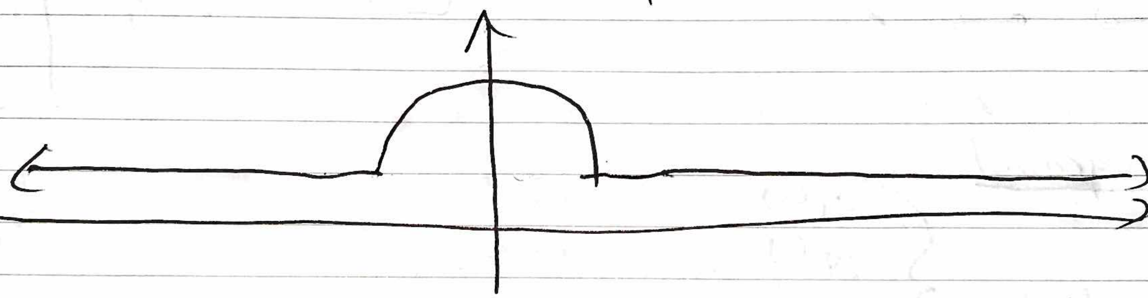
Straight lines above 1 & below -1

between -1 & 1,

$g = 1+\sqrt{1-x^2}$

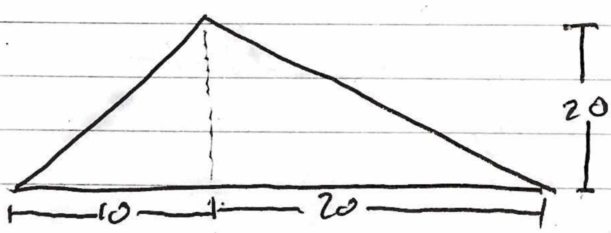
so $x^2+(g-1)^2 = 1$ ← radius 1 circle centered @ (0,1).

$\boxed{\text{upper half}}$



Final Example (HW 6, most people find hard).

Pizzeria Buonapetito makes a pizza of base width 30, & height 20 as shown below.

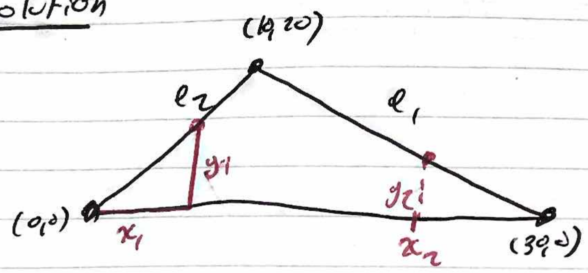


(a) Find formula for g = height @ x from left side, as a multipart fn ~~for $0 \le x \le 30$~~ State domain & range.

(b) I cut ~~[illegible]~~ @ x and take left side. Find area of my side as a multipart fn of x. State domain & range.

(c) I want exactly half the pizza. Where do I cut?

Solution



Notice 2 cases

① $x_1 \le 10$
then (x_1, y_1) on l_1

② $x_2 \ge 10$
then (x_2, y_2) on l_2

Case 1 $x \le 10$

Line w/ $(0,0)$ & $(10,20)$
slope $= 2$
intercept $= 0$

$\Big\}\ y = 2x$ is l_1

Case 2 $x \ge 10$

Line w/ $(10,20)$ & $(30,0)$.

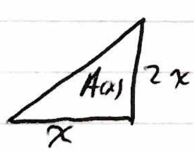
Slope $= -1$
Point $= (30,0)$

$y = -1(x-30)+0$
$= -x + 30$

$$h(x) = \begin{cases} 2x & 0 \le x \le 10 \\ 30 - x & 10 \le x \le 30 \end{cases}$$

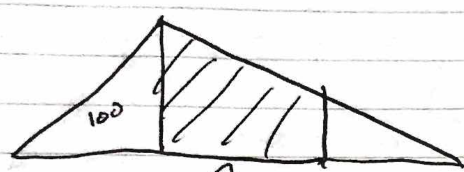
(b) Again, 2 cases

Case 1 $x \le 10$



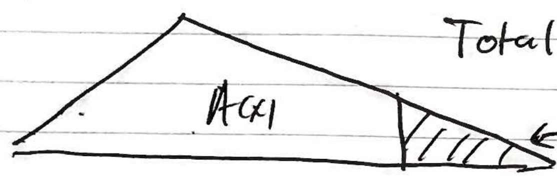
$A(x) = \frac{1}{2} x \cdot 2x = x^2$

$x \geq 10$



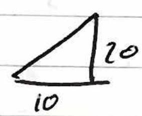
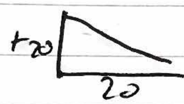
Use Trapezoids

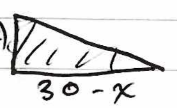
Or



Total − this*

Only requires triangles!

Total =  +  = 100 + 200 = 300

*This = 

$= \frac{1}{2}(30-x)(h(30-x))$

$= \frac{1}{2}(30-x)(30-x)$

$= \frac{1}{2}(30-x)(30-x)$

$= \frac{1}{2}x^2 - 30x + 450$

Total = 300 − (this)

$= -\frac{1}{2}x^2 + 30x - 150$

$$A(x) = \begin{cases} x^2 & 0 \leq x \leq 10 \\ -\frac{x^2}{2} + 30x + 150 & 10 \leq x \leq 30 \end{cases}$$

Day 7 (12)

Want $A(x) = 150$.

Case 1 If $x^2 = 150$ $x > 10$
$\Rightarrow x \notin D$,
so we wouldn't use that.

(also, know $x > 10$ b/c $A(10) = 100 < 150$)

So
$$-\frac{1}{2}x^2 + 30x - 300 = 0$$

~~gf~~ $x = 12.679$, ~~47.321~~ $\cancel{\in} D$

This one